ARR No. 4E31

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALLY ISSUED
May 1944 as
Advance Restricted Report 4E31

TORSION TESTS OF STIFFENED CIRCULAR CYLINDERS

By R. L. Moore and C. Wescoat
Aluminum Company of America

NACA

WASHINGTON

NACA ARR No. 4E31

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

TORSION TESTS OF STIFFENED CIRCULAR CYLINDERS

By R. L. Moore and C. Wescoat

INTRODUCTION

The design of curved sheet panels to resist shear involves a consideration of several factors: the buckling resistance of the sheet, the stress at which buckling becomes permanent, and the strength which may be developed beyond the buckling limit by tension-field action. Although some experimental as well as theoretical work has been done on the buckling and tension-field phases of this problem, neither of these types of action appears to be very well understood. The problem is of sufficient importance from the standpoint of aircraft design, it is believed, to warrant further experimental investigation. This report presents the results of the first series of torsion tests of stiffened circular cylinders to be completed in connection with this study at Aluminum Research Laboratories.

In some respects the shear problem for curved sheet panels is similar to that for flat panels. The buckling resistance depends not only upon curvature and the proportions of panel but also upon the restraint provided at the edges The strength which may be developed beyond the buckling limit depends upon the capacity of the sheet to transmit shear by diagonal tension and the resistance of the stiffener system to such tensile forces. The suddenness with which buckling generally occurs in a curved sheet would appear to make this action more determinate by experimental methods than is the case in flat sheet where the effects of eccentricities of loading usually obscure well-defined buckling phenomena. Tension-field action is more complex for curved panels, however, in that the stiffeners are subjected to combined bending and twist as well as axial compression.

Several solutions have been proposed for the problem of elastic shear-buckling in curved sheet panels. The ANC-5 Handbook (reference 1) indicates that this property may be determined by computing the critical stress for a flat panel of the same proportions and adding a factor depending upon curvature. This solution is similar in form to that proposed by Wagner and Ballerstedt (reference 2). Kromm (reference 3) has derived an approximate formula for the stability limit of infinitely long curved plate strips; while Ebner (reference 4) indicates that this property may be determined by formulas similar to those developed for curved panels under compression. Schapitz (reference 5) concludes that the shear-buckling action of curved sheet is largely unexplained. The latter reference presents the results of torsion tests of three stiffened circular cylinders and gives an analysis of tension-field action. Emphasis is placed upon the need for additional tests, not only to indicate shear-buckling characteristics, but to show the buckle patterns and stresses accompanying tension-field action.

OBJECT

The object of this investigation was to obtain, by means of torsion tests of stiffened circular cylinders, information on the behavior of curved sheet panels in shear. Four types of action have been considered:

(1) The shear-buckling resistance of cylinders having ring stiffeners only, for which the buckling limit is also the ultimate strength.

(2) The shear-buckling resistance of panels of cylinders having both longitudinal and ring stiffeners.

(3) The shear strength attainable beyond the buckling limit through the development of tension-field action in cylinders having both ring and longitudinal stiffeners.

(4) The permanent buckling characteristics of curved panels.

SPECIMENS

The specimens for these tests were circular cylinders, formed of 0.020- by 36- by 96-inch 24S-T sheet. The mean diameter was 30.08 inches and the over-all length 36 inches. Figure 1 gives the essential structural details. The ring stiffeners were made of 1/2- by 1/2-inch 24S-T square bars, shaped cold to approximate size in forming rolls. After forming, the rings were spliced and machined to obtain the required diameter. The longitudinal stiffeners were formed of 0.032-inch 24S-T sheet. Figure 1 shows the nominal dimensions. One of these stiffeners was used at the longitudinal seam of all specimens in order to prevent the waviness which might otherwise occur in a long thin lap joint having a large number of closely spaced rivets. The end bulkhead rings were made from 3/8-inch-thick steel plates.

Shear-buckling characteristics within the elastic range were investigated for 10 different sizes of curved sheet panels the dimensions and specimen number designations of which are shown in figures 5 and 11. All sizes of panels were obtained by varying the stiffener spacings of four different cylinders. Specimens 14, 15, 20, and 21, having the closest spacings of stiffeners, were the only ones tested to failure involving tension-field action.

PROCEDURE

Method of Loading

Figure 2 shows the loading fixture in which the torsion tests were made. This equipment consists of two similar structural steel frames having a depth of 3 feet 6 inches at the center and an effective lever arm of 12 feet 1/8 inch. One frame was held against rotation by anchoring it to the floor by means of bolts used in floor inserts; the other was rotated by means of loading screws so arranged as to pull in opposite directions at the ends. Spherically seated nuts were provided at each end of both loading screws to accommodate the movement accompanying the expected angles of twist.

The forces at the ends of the movable frame were determined by means of an aluminum alloy dynamometer link having a capacity of 5000 pounds. This force, applied in opposite directions at the ends of the loading frame, provided a maximum torque capacity of about 60,000 foot-pounds. Calibration of the dynamometer indicated a linear relation between load and deflection throughout the entire working range (0.236-in. deflection for 5000-lb load). Deflections were estimated by means of a dial indicator to the nearest 0.0001 inch, corresponding to a torque of about 25 foot-pounds.

The application of torque required two operators, one at each of the loading screws. Since the center of rotation was not fixed, it was necessary to provide some means of keeping the two ends of the specimen in the same relative position, otherwise some transverse bending as well as torque might have been applied. Figure 3 shows the bar which was mounted on the longitudinal axis of the specimens to serve as a reference for maintaining the proper position relative to the floor. By keeping a dial indicator at the rotating end in a position where it could be viewed by one loading-screw operator during the application of torque, it was possible to keep the vertical position of the center of rotation constant within 0.001 or 0.002 inch. Readings were taken at intervals at the fixed end of the specimen until it was demonstrated that for practical purposes these movements were negligible.

Although torque was applied in increments in all tests, it was generally not practical to attempt to apply definite predetermined values. The zero reading for each case was obtained with the specimen suspended loosely between the loading frames in order to eliminate accidental and unknown clamping torques. After the specimen was bolted in place, the loading arm was rotated until a dynamometer reading approximately equal to that desired was obtained, after which a final adjustment for cylinder position was made.

When buckling torques were being determined, it was necessary to watch the dynamometer deflection readings closely in order to catch the maximum values since the torques fell off as soon as buckling occurred. It was generally not possible from visual observation to predict when this buckling action would occur. In the specimens having ring stiffeners only,* an increase in

*Longitudinal stiffener used at seam in all specimens not considered effective in increasing shear-buckling resistance.

the angle of twist after buckling only resulted in a further decrease in torque. If the angle of twist was returned to zero, however, a torque reading approximately equal to the initial buckling value was obtained when the sheet snapped back to its original curved form. In specimens having 8 or 16 longitudinal stiffeners as well as ring stiffeners, this sudden buckling action and subsequent falling off of the torque occurred repeatedly, although at gradually increasing values, until all panels were buckled.

The specimens having ring stiffeners only were subjected to more than one loading in an effort to obtain more representative buckling values. Tests were repeated in some instances with the specimen rotated 180° with respect to its original position or turned end for end. Loadings were also tried in several instances with a number of end-connection bolts removed to minimize strains resulting from clamping the end bulkheads tightly to the loading frames. Although a range of buckling values was obtained by this procedure, there was no evidence that the buckling action in one test influenced the behavior in subsequent loadings. Buckling usually occurred in the same part of the cylinder in each test, regardless of its position in the loading frames.

Measurements

In addition to visual observations of the behavior of the cylinders and the determination of buckling and ultimate torques, measurements of over-all twist, radial deflections, and strains were made.

The twists were determined by means of a 10-inch level bar, used in the manner indicated in figure 3. This instrument, equipped with a 45-second bubble and a micrometer screw graduated in 0.001 inch, was sensitive to changes in slope of about one part in fifty thousand. The reference bars for these measurements, which also served to support the reference used for maintaining vertical position, were located on the inside face of the end bulkheads. The effective length of specimen was assumed to be $35\frac{5}{8}$ inches, or the distance between bulkhead centers.

Radial deflections were determined by means of a dial gage graduated in 0.001 inch and mounted on a sliding

pivot on the reference bar located at the center of the cylinders as shown in figure 3. This indicator could be used to traverse the entire inside surface of the specimens with the exception of a length of about 4 inches at each end. Rings with setscrews were used on both sides of the pivot to prevent sliding while readings at any one section were being taken. The procedure followed was to lay out a reference grid on one or more panels, including the stiffeners along the edge, which would provide sufficient data to indicate the progress and pattern of the buckling action of the sheet and stiffeners. Panels adjacent to the longitudinal seam of the specimens were not considered suitable for this purpose, although their action generally did not indicate any significantly different behavior.

Strains in the sheet were measured by means of Baldwin Southwark SR-4 type R-1 wire-resistance strain rosettes. These rosettes were used in pairs, one gage on each side of the sheet at each location. In some cases an attempt was made to obtain an approximate measure of the strain distribution in a typical panel by taking measurements at several different points as indicated in figure 4. In others, rosettes were located at the centers of panels only. These gages were attached by means of Duco household cement after the surface of the sheet had been roughened slightly with emery cloth (Aloxite No. 320) and cleaned with acetone.

Strain measurements on the longitudinal stiffeners were made by means of both SR-4 type A-1 wire-resistance strain gages and a 10-inch Whittemore strain gage. The Whittemore strain gage was more suited for measurements of extreme fiber stresses, although the gage length was too long in most cases to be sensitive to variations in bending along the length of the stiffeners. The wire strain gages were too wide to be cemented to the curved flanges of the longitudinal stiffeners and hence could be placed only on the sides where they were relatively insensitive to bending. A few strain measurements were taken on the ring stiffeners in the first tests by means of wire gages, but these determinations were not continued on all specimens.

Figure 4 shows the instrumentation used in the determination of strains by means of the wire-resistance gages. Unit strains were read directly in microinches by means of the Baldwin Southwark SR-4 strain indicator.

Computation of Stresses and Angles of Twist

Average shear stresses were computed from the relation

$$\tau = \frac{24T}{\pi D^2 t}$$

where

τ shear stress, psi

T torque, ft-lb

D mean diameter, in. (30.08)

and

t sheet thickness, in.

Angles of twist were computed from the relation

$$\theta = \frac{48TL}{\pi D^3 G t}$$

where

θ over-all twist, radians

L length, in. ($35\frac{5}{8}$)

and

G modulus of elasticity in shear, psi (3,950,000)

Measured strains were reduced to stresses using a modulus of elasticity of 10,500,000 psi and a Poisson's ratio of 1/3. The direction and the magnitude of the principal stresses in the sheet were determined from the rosette data by means of the usual biaxial stress relations.

The more complex formulas used for the computation of shear-buckling values and the stresses accompanying tension-field action are summarized in the appendix.

Auxiliary Tests of Stiffeners

Since the longitudinal stiffeners were formed sections, the actual dimensions differed slightly from the

nominal values indicated in figure 1. A representative value for cross-sectional area was determined by weighing several samples of known length. Moments of inertia about the principal axes and the location of the neutral axis in bending were determined from strain and deflection measurements made in bending tests under central concentrated loads. Compression tests also were made on individual stiffener sections ranging from 3 to 36 inches in length.

RESULTS AND DISCUSSION

Buckling of Cylinders Having Ring Stiffeners Only

Although numerous torsion tests of unstiffened circular cylinders have been reported (references 6 and 7), the behavior of specimens of this type provides a logical starting point for evaluating the effectiveness of stiffeners in increasing the shear-buckling resistance of curved sheet. Figure 5 shows typical torque-twist curves and indicates observed buckling ranges as well as theoretical buckling torques for different lengths of cylindrical section. The theoretical values were computed according to reference 6, assuming hinged edges. Data on measured wave lengths and angles of buckles are also included. Figures 6 to 10 show the nature and the extent of the buckling action produced.

As indicated by the shape of the torque-twist curves in figure 5, well-defined buckling values were obtained in all cases. Repeated tests of the same specimen did not give identical values of buckling torque, although the differences obtained were not large. Minimum torques in any one series of tests ranged from 83 to 98 percent of the maximum. Within the range of shear-resistant action before buckling, the torsional stiffness was in good agreement with that computed.

The maximum depths of buckles shown in figures 6 to 10 range from about 3/16 inch for the closest spacing of ring stiffeners to about 5/8 inch for the widest spacing. The fact that buckles of this depth could be produced repeatedly without any consistent or appreciable difference in the required torque indicates that the action was essentially elastic. It should not be assumed from these tests, however, that slight local permanent

sets are not a possibility at low shear-buckling stresses. When a large buckle such as that shown in figure 6 is formed rather violently, some part of the sheet may be subjected to a sharp enough curvature to leave a noticeable permanent crease, although the buckling resistance in subsequent tests may not be materially affected. Such an observation was made in the case of specimen 16 for a shear stress of only about 1100 psi.

Table I shows that the average observed shear-buckling stresses ranged from 77 to 93 percent of the theoretical values based on reference 6, assuming hinged edges. Some restraint was, of course, obtained at the end bulkheads and the intermediate ring stiffeners; although for the L/D ratios investigated, an assumption of clamped edges would have increased the theoretical buckling values only 10 to 12 percent. Part of the discrepancy between observed and theoretical buckling stresses undoubtedly may be attributed to local out-of-roundness or initial buckles in the cylinders. The difference between the observed buckling values for specimens 13 and 13A indicates that a more uniform distribution of shear stress and consequently a higher buckling value may be obtained for interior panels than for panels adjacent to the section of torque application. The observed angles of buckling ranged from about 3° to 10° greater than those computed according to reference 6, the differences being greatest for the closest spacing of ring stiffeners. The measured wave lengths were also somewhat greater than were computed.

In general, the results of these tests are in fair agreement with those reported by other investigators. The experimentally detérmined buckling torques given in reference 6 for cylinders without intermediate ring stiffeners averaged about 75 percent of the theoretical values for clamped edges with a minimum of 60 percent of the theoretical; whereas the observed values for the present tests averaged about 85 percent of the theoretical for hinged edges with a minimum in any one test of 70 percent of the theoretical. The method of computation given in ANC-5 (reference 1) is quite conservative in that it assumes that the minimum buckling resistance to be expected for average cylinders of the proportions considered will not exceed about 60 percent of the theoretical.

Buckling of Curved Panels

Figures 11 and 16 show the torque-twist curves and the buckling ranges or values observed for 10 different sizes of curved sheet panel. Although the torques for first buckling were fairly well defined, all like panels did not buckle simultaneously and it was necessary in most cases to determine the ranges over which this action occurred. The minimum buckling values for the different sizes of panel shown in figure 11 ranged from 74 to 89 percent of the maximum values. For the specimens having more than one spacing of ring stiffeners, the buckling of the largest panels was the only action definitely reflected by the torque-twist relations. Buckling ranges or values for the smaller panels were determined by visual observation.

Figures 12, 13, and 14 show the nature of the buckling action obtained in the panels where radial deflections were measured. The angles of first buckling were in every case slightly less than shown in figure 5 for specimens having the same spacings of ring stiffeners but without longitudinals, indicating that the longitudinals had a significant effect upon the buckling characteristics. The effect of the longitudinals probably would have been more pronounced had the spacings not been approximately equal to a multiple of the buckle wave lengths obtained without these stiffeners.

Table II summarizes the mean observed shear-buckling stresses for all sizes of panel and gives corresponding computed values obtained by several methods. Ratios of these observed to computed buckling stresses are given in table III. Of principal interest from the standpoint of design is the fact that the experimental buckling values exceed those computed according to ANC-5 (reference 1) by as much as 30 to 130 percent. The greatest differences are shown for the panels formed by a close spacing of ring stiffeners. The ANC-5 values were obtained by computing the buckling stresses for flat panels having the same proportions as those tested and adding a factor for curvature. Hinged edges were assumed, although for all but the closest spacings of ring stiffeners the assumption of partial or even complete edge fixity would not have altered the results appreciably.

The shear-buckling stresses computed according to references 3 and 4 are in much better agreement with observed values, although here again the greatest differences are shown for the closest spacings of ring stiffeners. The method of reference 3 was derived for infinitely long curved plate strips having simply supported edges. It was assumed that the buckling values for panels of the lengths considered were equal to the computed values for infinitely long panels multiplied by the ratio of the computed values for corresponding flat panels. In only two cases were the buckling values obtained by this procedure on the unsafe side. The method of reference 4 is more conservative and appears to offer a somewhat more practical basis for design. Failure to take proper account of edge restraining effects is probably partly responsible for its apparent shortcomings for close stiffener spacings. The importance of this factor is indicated by the fact that the theoretical buckling value for a 9-inch-long unstiffened cylindrical section of the proportions tested is only about 12 percent higher for fixed than for hinged edges; whereas for a 2-inch-long section the corresponding increase in buckling value is about 40 percent.

In the tests of the specimens having ring stiffeners only it will be recalled from table I that the observed buckling stresses averaged about 85 percent of the theoretical based on reference 6, assuming hinged edges. Table III shows that the mean observed buckling stresses for different lengths of cylindrical sections having 8 longitudinal stiffeners were from 92 to 106 percent of the theoretical values for the same lengths of unstiffened cylinders; whereas the buckling values for cylinders having 16 longitudinal stiffeners were from 26 to 56 percent greater than the theoretical for the corresponding unstiffened cylinders. Figure 15 illustrates these relations between shear-buckling stress, L/D ratio, and the number of longitudinal stiffeners. It appears from these tests that the buckling stress for an unstiffened cylinder may be increased about 20 percent by adding 8 equally spaced longitudinal stiffeners or increased about 60 percent by adding 16 equally spaced stiffeners.

Although the relative merits of different sizes of longitudinal stiffeners have not been investigated, the particular section chosen for these tests was adequate as far as buckling resistance was concerned in that it

remained essentially straight after buckling of the sheet panels. How much smaller the stiffeners might have been and still accomplish the same result is, of course, not known. It is instructive to point out that as far as the shear-buckling resistance of the panels was concerned, the material in the stiffeners might have been used somewhat more effectively if it had been added uniformly to the cylinder wall thickness. According to computations the shear-buckling resistance would have been increased about 40 percent by utilizing the material of 8 longitudinals or 80 percent by utilizing that of 16 stiffeners. These increases in buckling resistance are slightly greater than obtained by the use of stiffeners.

Tension-Field Action

Tests to failure, involving the application of torques of from four to six times the values required to produce first panel buckling, were made on only four cylinders. Figure 16 shows complete torque-twist curves for these specimens and indicates buckling ranges and maximum torques.

The action of stiffened circular cylinders in the tension-field range is so complex that no attempt has been made from these few tests to formulate general conclusions regarding such behavior. Some indication of the principal elements of the problem may be obtained, however, from a consideration of the torsional stiffness after buckling, the buckle patterns and sheet stresses, the stresses and deflections produced in the stiffeners, and the ultimate torsional strengths.

Torsional stiffness. - The torque-twist curves in figure 16 indicate four distinct stages in the behavior of the cylinders: (1) the range of shear-resistant action before buckling, for which the torsional stiffness may be predicted closely on the assumption of pure shear; (2) the buckling range, where there is a marked decrease in the torsional stiffness as the sheet is being stretched to where it is capable of transmitting shear by diagonal tension; (3) the tension-field range in which the shear in excess of the buckling value is carried principally by tension in the sheet and an approximately linear relation between torque and twist is again obtained;

and (4) ultimate failure, which may occur either by fracture of the sheet or by collapse of some element of the stiffener system.

Torsional stiffness in the range after buckling appears to be dependent not only upon sheet thickness and proportions of panel but also upon the properties of the stiffeners. The use of two or more quite different ring-stiffener spacings in all specimens except specimen 15 made it impossible to evaluate the stiffness of the different sizes of panels from measurements of over-all twist only. Ratios of the rates of twist after buckling to those before buckling were approximately as follows:

Specimen 14 - 3.9
Specimen 15 - 4.1
Specimen 20 - 3.2
Specimen 21 - 2.8

The fact that specimen 21 was the stiffest in the tension-field range is not surprising in view of the close spacings of stiffeners, although an explanation for the relative stiffnesses of specimens 14, 15, and 20 is not apparent from inspection. The large decrease in torsional stiffness after buckling is evidence of several factors of importance in an analysis of tension-field action: (1) the angle and distribution of the diagonal tensile stresses in each panel, (2) the stretching of the sheet between longitudinals from an arc to a position approaching the chord, and (3) the radial deflections of the longitudinal and ring stiffeners.

Buckle patterns. - Figures 12, 13, and 14, previously referred to, illustrate the buckling characteristics observed for torques covering the greater part of the tension-field range investigated. The position and angle of buckling, which it should be noted are not constant with varying torques, are indicative of the paths of the principal diagonal tensions. The torque-radial deflection curves shown in figure 17 gives an indication of the progress of this buckling. In those cases for which the number of buckles and the location of the points of maximum deflection remained fairly constant, the curves indicate well-defined buckling action within the range of critical torques selected. In those cases where there was a shift in the buckle pattern, however,

the break or knee in the torque-deflection curves which is indicative of buckling was less pronounced.

Sheet stresses. - Figures 18 to 21 show the torque stress relations obtained from the strain rosette measurements. In all cases the stresses given are the averages of the measured values on the two sides of the sheet. In general, the measured stresses before buckling were reasonably consistent with those computed on the assumption of pure shear. The diagonal tensions and compressions were approximately equal to the agerage shears and were inclined 45° to the longitudinal axis of the specimens. The buckling torques indicated by these stress determinations were within the ranges determined from the twist measurements. After buckling, the diagonal tensions continued to vary linearly with torque but their magnitudes increased to two or more times the average shears and there was a marked shift in their direction. The corresponding diagonal compressions varied little in most cases from their values at the buckling torques.

In view of the type of buckling shown in figures 12, 13, and 14, it was not expected that a very accurate measure of the maximum sheet stresses accompanying tension-field action could be obtained. In cases where stresses were measured at more than one location in the same panel, the values at the center were the highest. An exception to this rule probably would have been found had strain measurements been taken on specimen 19 (see buckle pattern in fig. 14). Local bending stresses, which govern permanent buckling characteristics, were obviously much higher than any of the average values indicated but the locations and magnitudes of these critical stresses were of course unknown.

The irregularities shown in certain of the torque-stress relations for specimens 15 and 21, in the range just after buckling, may apparently be attributed largely to the effect of shifting buckle patterns. For these cases, the computed angles of maximum tensions ranged from about 45° before buckling, to values as low as 13° after buckling. The fact that these angles were not consistent with the wave angles shown in figure 12 suggests that the stresses measured were not the maximum tensions developed in the panels. In specimens 14 and 20, where there was less shift in the buckle patterns

relative to the strain rosettes, a better agreement between observed angles of buckling and computed angles of maximum tensions was obtained.

Stiffener stresses and deflections. - Figures 22 to 25 indicate the manner and extent to which the stiffeners participated in the development of tension-field action after buckling. The relations between torque and average compressive stress in the longitudinal stiffeners were approximately linear. In general, the stresses indicated by the wire-resistance strain gages (13/16-in. gage lengths) attached to the sides of the longitudinals were in fair agreement with the averages of the extreme fiber stresses determined by means of the 10-inch Whittemore strain gage. Although the use of more than one ring-stiffener spacing in all specimens except specimen 15 resulted in different degrees of tension-field action in different panels and consequently variations in average compression along the lengths of the longitudinals, it is believed that a fairly satisfactory measure of the maximum compressive forces developed in these members was obtained.

It is apparent from the measured radial deflections and the stress-distribution diagrams included in figures 22 and 25 that the longitudinal stiffeners were subjected to bending as well as to axial compression. It was not possible from the measurements made, however, to determine the maximum intensity of these bending stresses. The location of the wire-resistance gages on the sides of the stiffeners made them insensitive to bending and the Whittemore readings gave only average stresses over 10-inch gage lengths. The amount of integral action developed between the stiffeners and the sheet to which they were attached was also an uncertain factor.

Figure 22 gives the results of the few strain measurements made on the ring stiffeners of specimen 14. These stresses were not only small but appeared to be influenced by local bending effects which could not be fully evaluated. The observed radial deflections give the most direct measure of the over-all action of the ring stiffeners. These deflections, as well as those of the longitudinal stiffeners, did not vary linearly with torque but increased at an increasing rate as the sheet was "stretched" between longitudinals. The greater

the departure of the sheet from its original curved form, the greater the radial component of the sheet tension acting on the stiffeners. The average compressive stresses developed in the ring stiffeners may be estimated by assuming that the deflections were the result of a uniformly distributed radial pressure, since there was no significant difference between deflections measured at or midway between longitudinals. The maximum deflection of 0.025 inch shown in figure 25 for the center stiffiner of specimen 21 corresponds to an average compressive stress of about 17,800 psi.

Analysis of tension-field action. - Reference 5 indicates the factors to be considered in an analysis of tension-field action in stiffened circular cylinders and gives formulas for computing the principal stresses involved. It is apparent from an attempt to check the behavior observed in these tests against that computed by the proposed methods, however, that certain aspects of the problem are still not well understood.

Table IV shows a comparison between measured sheet and stiffener stresses and values computed on the assumption of complete tension-field action. The angles of principal tension used were assumed to coincide with the observed angles of buckling. The fact that the ratios of measured to computed stresses for the longitudinal stiffeners are less than unity indicates that partial rather than complete tension-field action should have been assumed. The majority of the ring stiffener stresses, however, exceed the computed values, the differences in the case of specimen 21 being considerable. The maximum measured sheet stresses are in fair agreement with those computed.

As indicated in reference 5, the use of the rather complex formulas proposed for incomplete tension-field action requires the evaluation of several experimentally determined factors. Except for the degree of tension-field action indicated by the ratios of measured to computed longitudinal stiffener stresses shown in table IV, these tests have not contributed materially toward such an evaluation. It may be shown that the angles of buckling observed for the different panels are not consistent with the so-called "wrinkling" factor for complete tension-field action. Furthermore, the use of more than one spacing of ring stiffeners in all specimens except 15 precluded the possibility of evaluating this factor from

measurements of over-all twist. Even in the case of specimen 15, where a determination of this wrinkling factor could be made, the computed angle of principal tension was not in agreement with the angle of buckling observed. Until considerably more data relating to tension-field action in stiffened cylinders are available, there appears to be little basis for making other than the generally conservative assumption of complete tension-field action.

Ultimate torsional strengths. - Table V gives the average shear and the estimated maximum tensile and compressive stresses developed in the four specimens loaded to failure. In three cases failure occurred by collapse of the longitudinal stiffeners as shown in figures 26, 28, and 29; in the fourth the sheet fractured through the connections to one of the end bulkheads as shown in figure 27. The fourth test was the one which indicated the need for providing relatively high shear-resistant panels adjacent to the end bulkheads to cushion tension-field effects. The effectiveness of this procedure is indicated by the fact that the average shear stress developed in specimen 21 at failure was about 40 percent higher than that developed in specimen 15, having the same stiffener system except for the cushion panels at the ends.

The longitudinal stiffeners of specimens 14 and 21 collapsed at approximately the same estimated average compressive stress, indicating that the length between ring stiffeners rather than the number of longitudinals was the controlling factor. The ring-stiffener spacing at the center of specimen 20 was 50 percent greater than in specimens 14 and 21, and consequently the longitudinals collapsed at a lower stress. The buckling stresses developed in the longitudinal stiffeners of these three cylinders, for ring-stiffener spacings of 9 inches and 13.5 inches, were in close agreement with the stresses developed by the same lengths of individual stiffener section, tested as flat-end columns. Figure 30 shows a comparison of these data. Failure in the column tests occurred by twisting rather than by flexure, which accounts for the considerable difference shown between the test values and the computed Euler column curve.

Permanent sets. - Although no systematic effort to determine relations between buckling and first permanent set characteristics was made in these tests, some information relating to this aspect of behavior was obtained.

The photographs in figures 26, 28, and 29 were taken with considerable torque on the cylinders to emphasize the buckle patterns. Except in the vicinity of the longitudinal stiffeners which collapsed, the greater part of these buckles in the sheet disappeared when the torque was removed. Figure 27 shows practically no evidence of permanent buckling in the center panels of specimen 15 after the application of an average shear of 14,400 psi, accompanied by sheet deflections as high as 0.35 inch. The torque-twist curve shown for this specimen in figure 16, however, indicates considerable overall permanent set for a torque equal to only about 80 percent of the maximum. Measured deflections for specimen 14 indicated that depths of buckles of about 0.25 inch were produced by an average shear of 5100 psi with no appreciable set, although subsequent loading of this specimen showed a reduction in mean buckling value of about 8 percent. It appears from these data that tests to determine losses in buckling resistance may provide a more sensitive measure of significant permanent set than direct measurements of permanent sheet deflection.

SUMMARY AND CONCLUSIONS

The torsion tests of stiffened circular cylinders described in this report are the first of a series to be completed in an experimental investigation of the shear-buckling resistance and strength of stiffened curved sheet. Although a number of observations of interest have been made regarding the behavior of this particular group of 0.020-inch thick 24S-T cylinders, additional tests are in progress which should be considered before an attempt is made to formulate general conclusions. The most significant results obtained thus far may be summarized as follows:

1. The shear-buckling stresses observed for the cylinders having ring stiffeners only averaged about 85 percent of the theoretical values computed according to Donnell, (reference 6), assuming hinged edges. This average is about 40 percent higher than the minimum to be expected from average cylinders of these proportions, according to ANC-5 (reference 1). Although the results of these tests are in fair agreement with those previously reported, their value in this investigation lies in the fact that they provide a basis for judging the relative effectiveness of longitudinal stiffeners in increasing the shear-buckling resistance of curved sheet.

2. Table II gives mean observed shear-buckling stresses for 10 different sizes of curved sheet panels. These stresses were based on buckling phenomena which for the most part were well defined by the torque-twist relations shown in figures 11 and 16. Variations from the mean observed buckling value for any one size of panel did not exceed 15 percent.

3. The observed shear-buckling stresses given in table II are from 30 percent to 130 percent higher than computed according to ANC-5, assuming hinged edges. Computed values based on a modification of the solution given by Kromm (reference 3) for infinitely long curved panels, or upon the method proposed by Ebner (reference 4) are in much better agreement with test values. These methods should receive further consideration as a possible basis for design.

4. A comparison of the shear-buckling stresses observed for different lengths of cylindrical section, with and without longitudinal stiffeners, and the theoretical values for unstiffened sections indicates that the buckling resistance of a given cylinder may be increased about 20 percent by adding 8 equally spaced longitudinal stiffeners, or about 60 percent by adding 16 equally spaced longitudinals.

5. The results of the tests carried to ultimate failure permit a qualitative if not a very accurate quantitative analysis of the action involved in the tension-field range beyond buckling. Approximately linear relations were observed between torque and over-all twist for the greater part of the tension-field ranges, as shown in figure 16, although the rates of twist were three to four times those measured in the shear-resistant range before buckling. Linear relations were likewise observed between torque and average measured stresses in the sheet and longitudinal stiffeners, as shown in figures 18 to 25. The average compressive stresses in the ring stiffeners, based on measured radial deflections, and the bending stresses in the longitudinals showed a definite tendency to increase at a faster rate than the torques.

6. An analysis of tension-field action in stiffened circular cylinders subjected to twist involves a consideration of the buckling resistance of the sheet, the magnitude and direction of the principal tensile stresses produced in the sheet after buckling, the contraction of the sheet between longitudinals as a result of buckling,

and the size and spacing of the stiffeners. Schapitz (reference 5) has indicated the manner in which these factors may be recognized, but except for an assumed case of complete tension-field action, his formulas require substitution of experimental constants not yet evaluated.

7. Table IV gives a comparison between measured sheet and stiffener stresses and values computed for complete tension-field action according to Schapitz (reference 5). The ratios of average measured to computed stresses in the longitudinals indicate that incomplete rather than complete tension-field action should have been assumed, although the results of the other stress measurements are not generally consistent with this observation.

8. Three of the four specimens loaded to the limit of the tension-field range failed by collapse of the longitudinal stiffeners. The average compressive stresses developed for ring-stiffener spacings of 9 inches and 13.5 inches were approximately the same as the strengths developed by the same lengths of individual stiffener sections tested as flat-end columns. The fourth specimen failed by fracture of the sheet through the connections to one end bulkhead.

9. Relations between average shear stresses and the maximum deflections and bending stresses which may be developed in sheet panels of different proportions after buckling have not been sufficiently well defined to make possible accurate predictions regarding permanent buckling characteristics. If a structure can be loaded repeatedly without loss in buckling resistance, it can be assumed, of course, that the action is elastic. Tests to determine allowable load limits for elastic buckling appear to provide a more sensitive measure of permanent set than direct measurements of permanent sheet deflection.

Aluminum Research Laboratories,
Aluminum Company of America,
New Kensington, Pa., March 13, 1944.

APPENDIX

SUMMARY OF FORMULAS

Notation

τ	shear-stress for applied torque, psi
τ_o	shear-buckling stress, psi
t	sheet thickness, in.
D	mean diameter, in.
r	mean radius, in.
L	length, in.
b	arc length of panel, in.
a	other dimension of panel, in.
σ	maximum diagonal tension after buckling, psi
α	angle of maximum diagonal tension, deg
σ_x	average compression in longitudinal stiffeners, psi
σ_y	average compression in ring stiffeners, psi
F_x	area of longitudinal stiffener, sq in.
F_y	area of ring stiffener, sq in.
E	modulus of elasticity, psi, (10,500,000)
μ	Poisson's ratio (1/3)

Reference 1 (ANC-5)

For unstiffened circular cylinders, where $\frac{L^2}{tD} > 20$

$$\tau_o = K \frac{E}{\left(\frac{D}{t}\right)^{5/4} \left(\frac{L}{D}\right)^{1/2}}$$

where

$K = 0.75$ for hinged edges

For curved panels

$$\tau_o = KE\left(\frac{t}{b}\right)^2 + K_1 E \frac{t}{r}$$

where

K buckling factor for corresponding flat sheet panel depending on ratio of a/b

and

$K_1 = 0.1$

Reference 3 (Kromm)

For infinitely long curved plate strips having hinged edges (central angle $\leqq 43^\circ$)

$$\tau_o = 1.67\, E \frac{t}{b} \sqrt{\frac{t}{r}}$$

when $\frac{b}{t}\sqrt{\frac{t}{r}} > 4.3$ (See reference 3 for other cases.)

For the panel lengths considered, values of τ_o determined by the above relation were multiplied by the ratio of the computed buckling stress for a flat panel of the same proportions to that for an infinitely long flat panel having the same short dimension.

Reference 4 (Ebner)

For curved panels

$$\tau_o = \sqrt{\tau_R{}^2 + \left(\frac{\tau_P}{2}\right)^2} + \frac{\tau_P}{2}$$

where

τ_R shear-buckling stress for unstiffened cylinder

τ_P shear-buckling stress for corresponding flat panel

Reference 5 (Schapitz)

For complete tension-field action after buckling

$$\sigma = \frac{\tau}{\sin \alpha \cos \alpha} - \tau_o$$

$$\sigma_x = \frac{bt}{F_x} \tau_o \left(\frac{\tau}{\tau_o} \cot \alpha - 1 \right)$$

$$\sigma_y = \frac{at}{F_y} \tau_o \left(\frac{\tau}{\tau_o} \tan \alpha - 1 \right)$$

Reference 6 (Donnell)

For unstiffened circular cylinders having hinged ends where $\frac{L^2 t}{D^3} < 5.2$

$$\tau_o = \frac{Et^2}{(1-\mu^2)L^2} \left[2.8 + \sqrt{2.6 + 1.4 \left(\sqrt{1-\mu^2} \frac{L^2}{tD} \right)^{3/2}} \right]$$

REFERENCES

1. Anon.: Strength of Aircraft Elements. Army-Navy-Civil Committee on Aircraft Design Criteria (ANC-5), 1942.

2. Wagner, H., and Ballerstedt, W.: Tension Fields in Originally Curved, Thin Sheets during Shearing Stresses. NACA TM No. 774, 1935.

3. Kromm, A.: The Limit of Stability of a Curved Plate Strip under Shear and Axial stresses. NACA TM No. 898, 1939.

4. Ebner, H.: The strength of Shell Bodies - Theory and Practice. NACA TM No. 838, 1937.

5. Schapitz, E.: The Twisting of Thin-Walled, Stiffened Circular Cylinders. NACA TM No. 878, 1938.

6. Donnell, L. H.: Stability of Thin-Walled Tubes under Torsion. NACA Rep. No. 479, 1933.

7. Lundquist, E. E.: Strength Tests on Thin-Walled Duralumin Cylinders in Torsion. NACA TN No. 427, 1932.

TABLE I

TORSIONAL STRENGTHS OF 24S-T CYLINDERS HAVING RING STIFFENERS ONLY

Mean Diameter, D = 30.08 in.
Overall Length = 36 in.
Sheet Thickness = 0.020 in.

Specimen No.	Length of Buckled Section, L, in.†	L/D	Shear-Buckling Stresses, psi		
			Average Observed	Theoretical*	Observed / Theoretical
16	35.62	1.18	1 110	1 325	0.84
17	27.00	0.90	1 280	1 520	0.84
18	13.50	0.45	1 710	2 210	0.77
13	8.81	0.29	2 310	2 870	0.81
13A	9.00	0.30	2 570	2 770	0.93

† Center to center of ring stiffeners (See Fig. 5).

* Based on Reference 6, assuming hinged edges.

TABLE III

COMPARISON OF OBSERVED AND COMPUTED SHEAR-BUCKLING STRESSES FOR CURVED 24S-T SHEET PANELS GIVEN IN TABLE II

Specimen No.	Dimensions of Panels, in.	Ratio of Mean Observed to Computed Shear-Buckling Stresses Based On:			
		ANC-5	Ref. 3	Ref. 4	Ref. 6
	8 Longitudinal Stiffeners, 11.81 in. apart				
19	0.020 x 13.50 x 11.81	1.30	1.17	0.91	0.95
14	0.020 x 9.00 x 11.81	1.68	1.09	1.00	1.06
14	0.020 x 4.31 x 11.81	1.58	0.43	0.80	0.92
	16 Longitudinal Stiffeners, 5.90 in. Apart				
20	0.020 x 13.50 x 5.90	1.39	1.10	1.12	1.30
15	0.0195 x 8.81 x 5.90	1.60	-	-	1.26
15	0.0195 x 9.00 x 5.90	1.71	1.24	1.13	1.37
21	0.0205 x 9.00 x 5.90	1.84	-	-	1.42
21	0.0205 x 6.75 x 5.90	1.99	1.30	1.21	1.39
20	0.020 x 4.31 x 5.90	2.31	1.25	1.27	1.49
21	0.0205 x 2.06 x 5.90	2.02	0.71	1.50	1.56

TABLE V

TORSIONAL STRENGTHS OF 24S-T CYLINDERS HAVING BOTH RING AND LONGITUDINAL STIFFENERS

Mean Diameter = 30.08 in.
Overall Length = 36 in.

Specimen No.	Sheet Thickness, in.	Number of Longitudinal Stiffeners	Ring-Stiffener Spacing for Center Panels, in.	Maximum Torque, ft-lb	Stresses at Failure, psi			Location of Failure
					Average Shear	Maximum Tension in Sheet	Average Compression in Longitudinals	
14	0.020	8	9	28 600	12 100	33 500	34 000	Longitudinals
15	0.0195	16	9	33 200	14 400	35 000	20 000	Sheet
20	0.020	16	3.5	36 800	15 500	48 500	23 000	Longitudinals
21	0.0205	16	9	49 000	20 200	45 500	34 500	Longitudinals

TABLE II

SHEAR-BUCKLING STRESSES FOR PANELS OF 24S-T CYLINDERS HAVING BOTH RING AND LONGITUDINAL STIFFENERS

Mean Diameter = 30.08 in.
Overall length = 36 in.

Specimen No.	Dimensions of Panels, in.	Shear-Buckling Stresses, psi				
		Mean Observed	Computed			
			ANC-5	Ref. 3	Ref. 4	Ref. 6*
	8 Longitudinal Stiffeners, 11.81 in. apart					
19	0.020 x 13.50 x 11.81	2 110	1 620	1 810	2 320	2 210
14	0.020 x 9.00 x 11.81	2 950	1 750	2 700	2 960	2 770
14	0.020 x 4.31 x 11.81	4 130	2 610	9 700	5 145	4 510
	16 Longitudinal Stiffeners, 5.90 in. apart					
20	0.020 x 13.50 x 5.90	2 870	2 070	2 600	2 570	2 210
15	0.0195 x 8.81 x 5.90	3 370	2 110	-	-	2 680
15	0.0195 x 9.00 x 5.90	3 590	2 100	2 880	3 170	2 680
21	0.0205 x 9.00 x 5.90	4 120	2 240	-	-	2 910
21	0.0205 x 6.75 x 5.90	4 740	2 380	3 650	3 900	3 400
20	0.020 x 4.31 x 5.90	6 750	2 920	5 400	5 300	4 510
21	0.0205 x 2.06 x 5.90	13 900	6 890	19 500	9 300	8 900

* Theoretical for cylinders having lengths equal to ring-stiffener spacings but without longitudinals.

TABLE IV

COMPARISON OF MEASURED AND COMPUTED STRESSES WITHIN THE TENSION-FIELD RANGE
STIFFENED CIRCULAR 24S-T CYLINDERS

Specimen No.	Torque, ft-lb	Avg. Compression in Longitudinals, psi			Avg. Compression in Rings, psi			Maximum Tension in Sheet, psi		
		Measured*	Computed	Meas./Comp.	Measured†	Computed	Meas./Comp.	Measured**	Computed	Meas./Comp.
14	12 000	10 000	21 600	0.46	-	-	-	12 000	9 400	1.28
	20 900	22 500	35 000	0.64	1 400	2 000	0.70	22 000	16 400	1.34
	28 000	33 000	49 900	0.66	3 500	3 400	1.03	32 000	23 000	1.39
15	11 600	8 500	12 300	0.69	-	-	-	8 500	9 800	0.87
	20 000	12 000	17 400	0.69	2 100	1 100	1.91	15 000	16 100	0.93
	30 800	19 000	27 000	0.70	4 200	3 500	1.20	26 000	25 600	1.01
20	13 900	9 500	21 700	0.44	-	-	-	13 000	15 500	0.84
	22 200	14 000	33 100	0.42	-	1 000	-	24 000	23 800	1.01
	27 800	18 500	42 500	0.44	-	2 000	-	33 500	30 700	1.09
21	16 800	10 000	12 400	0.80	-	-	-	9 500	11 500	0.82
	22 200	14 000	16 900	0.83	2 800	1 400	2.00	14 500	16 000	0.91
	30 200	20 000	25 000	0.80	5 600	2 900	1.93	23 000	23 100	0.99
	41 600	29 000	37 300	0.78	17 500	5 100	3.44	36 500	33 400	1.09

All computed stresses based on Reference 5, assuming complete tension-field action.

* Based on average strains measured over 10-in. gage lengths near center of stiffeners.
** Based on average strains measured at centers of panels.
† Based on measured radial deflections.

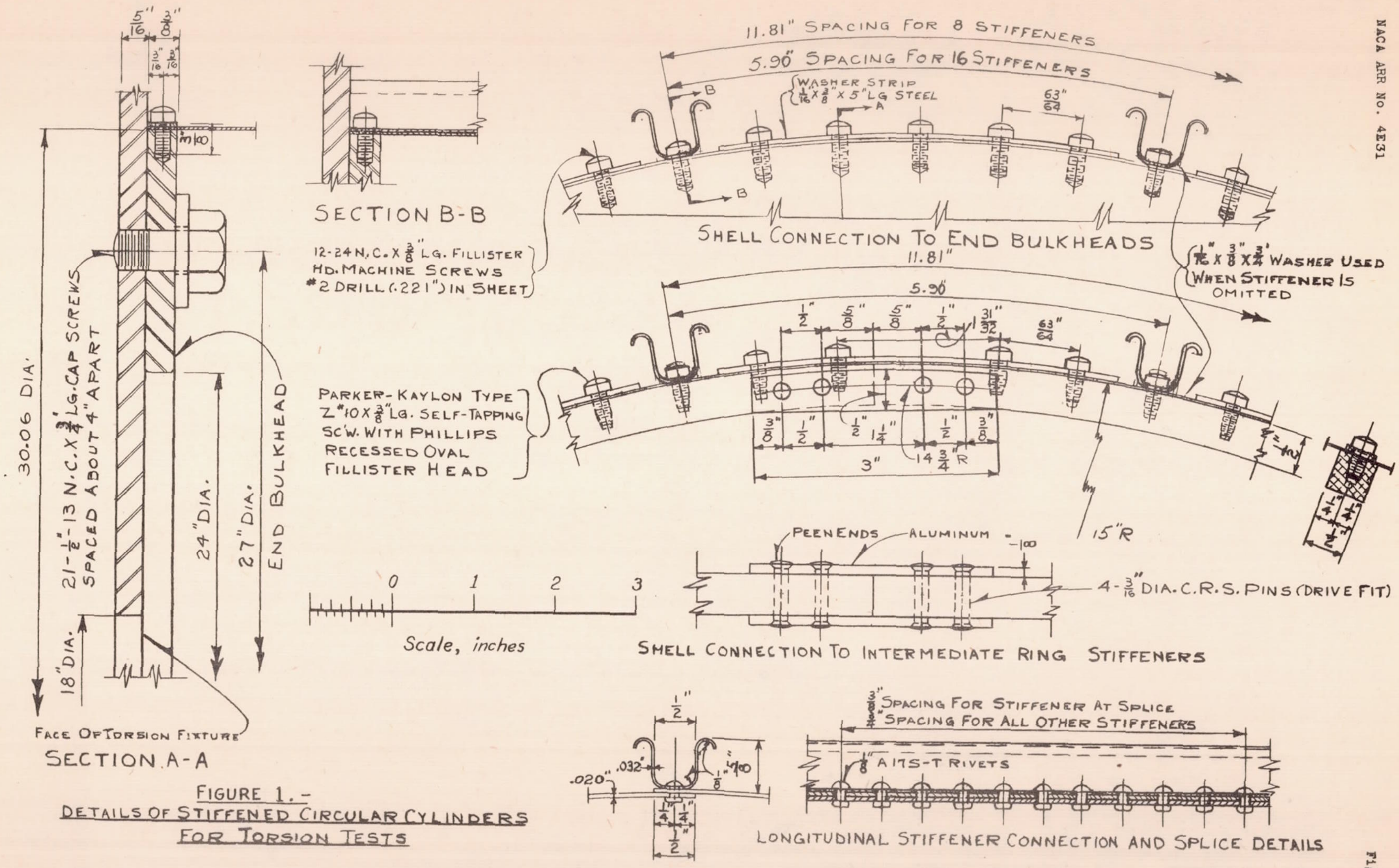


FIGURE 1. - DETAILS OF STIFFENED CIRCULAR CYLINDERS FOR TORSION TESTS

w-89

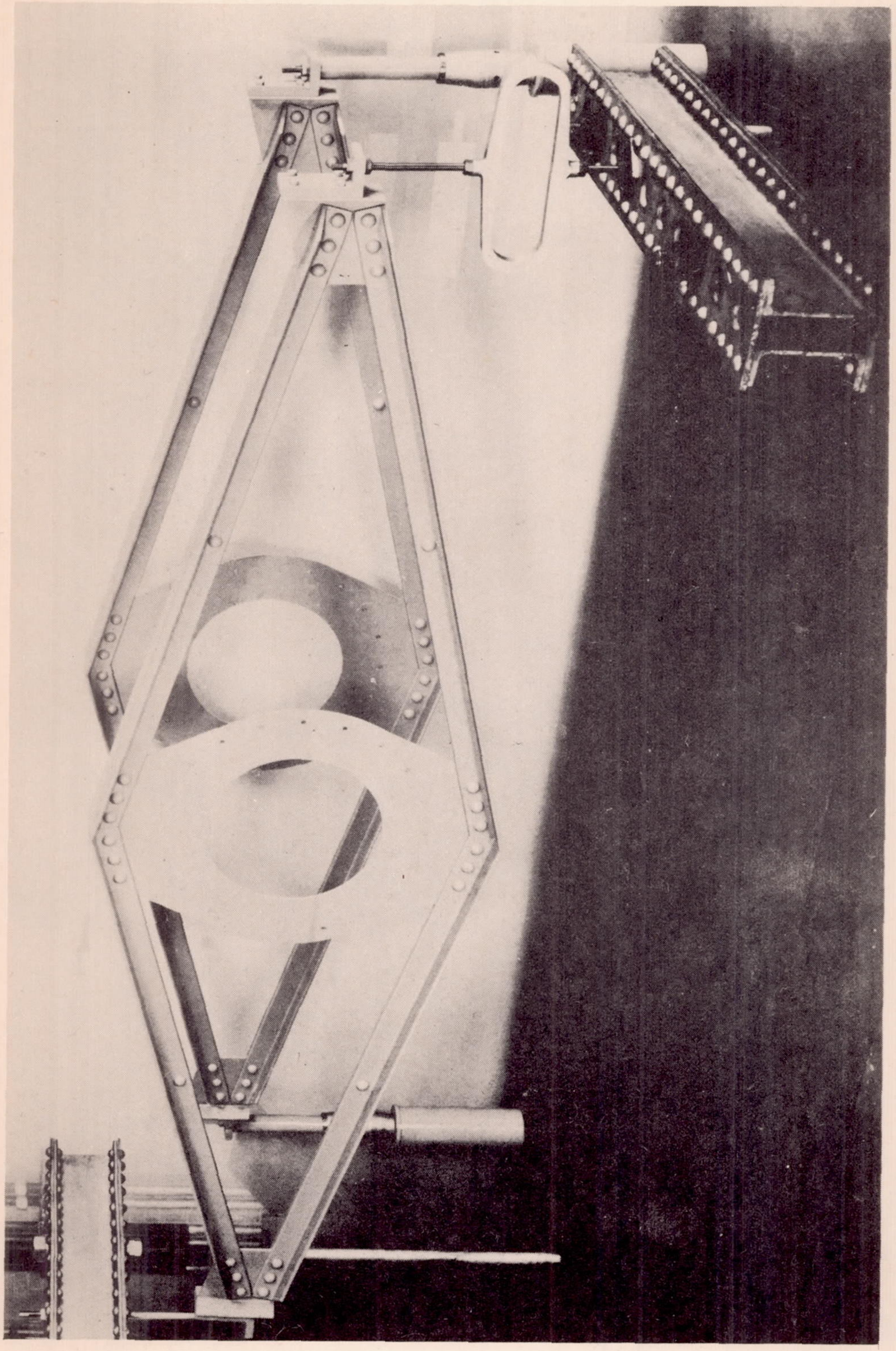


Figure 2. - 60,000 ft/lb capacity torsion loading fixture.

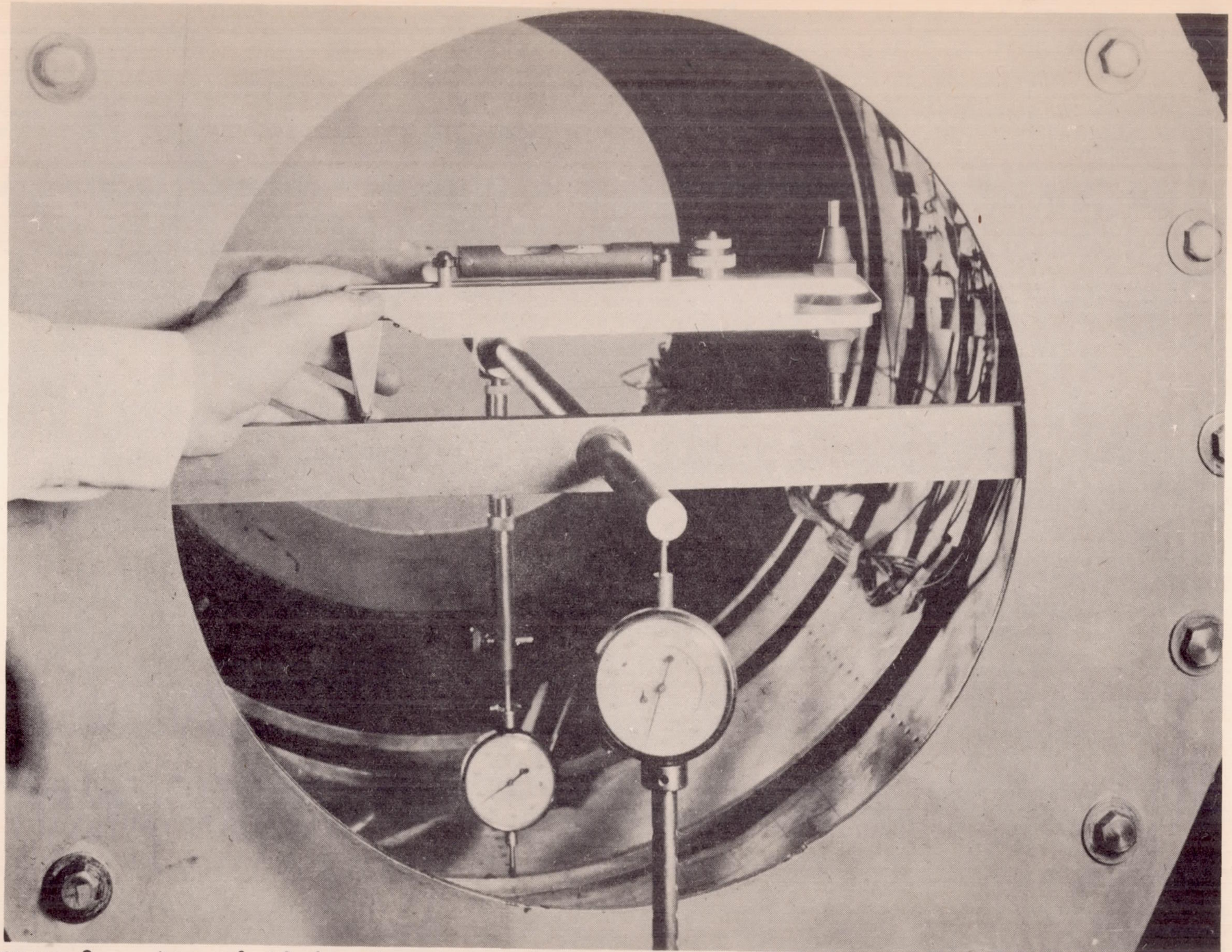


Figure 3. - Use of 10-in. level bar for measuring twist. Dial indicator inside cylinder used for measuring radial deflections. Dial indicator in foreground used to keep rotating end of cylinder in fixed position relative to the floor.

w-89

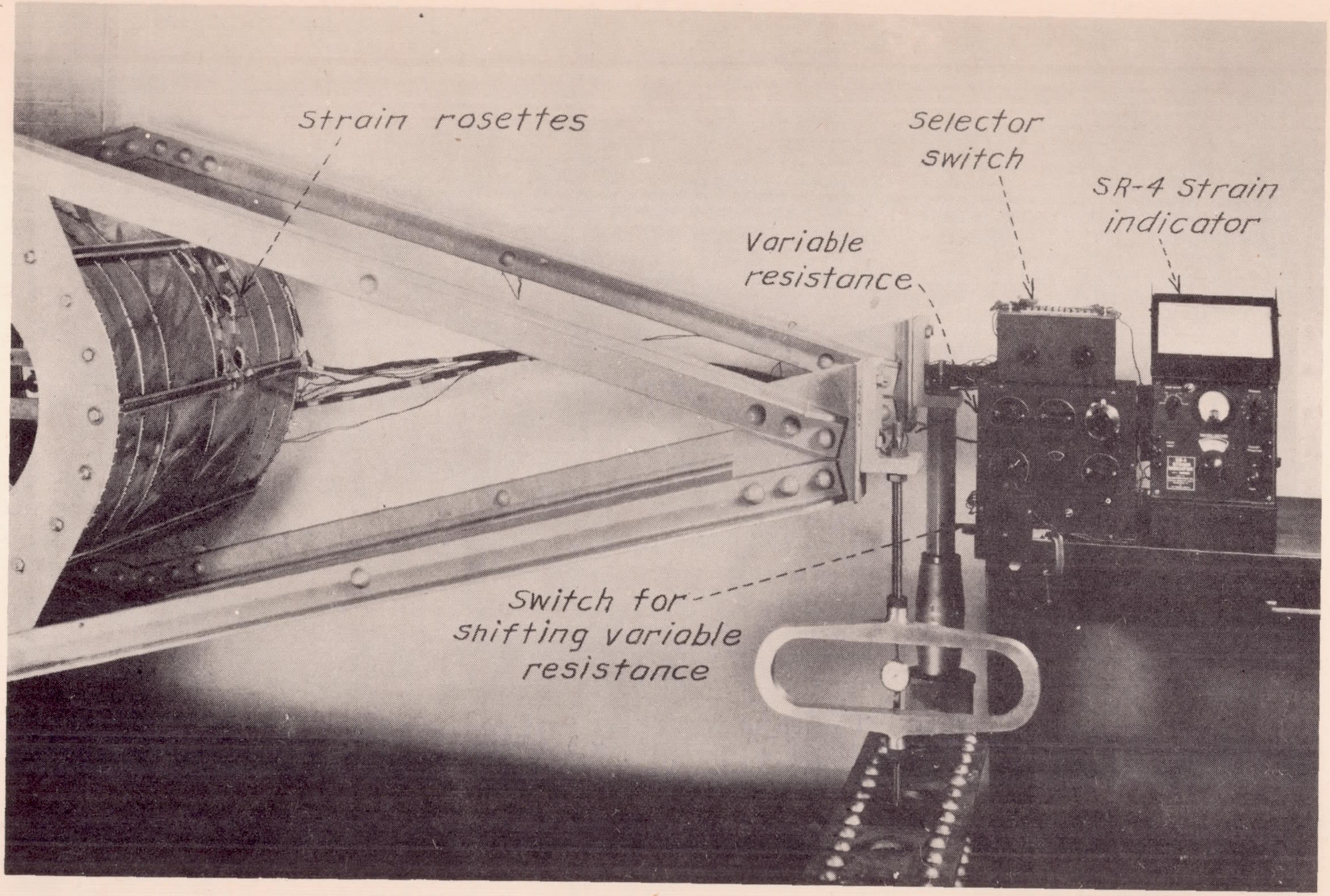


Figure 4. - Apparatus used in measuring strains by means of Baldwin Southwark SR-4 wire-resistance gages. Cylinder shown is specimen No. 14.

w-89

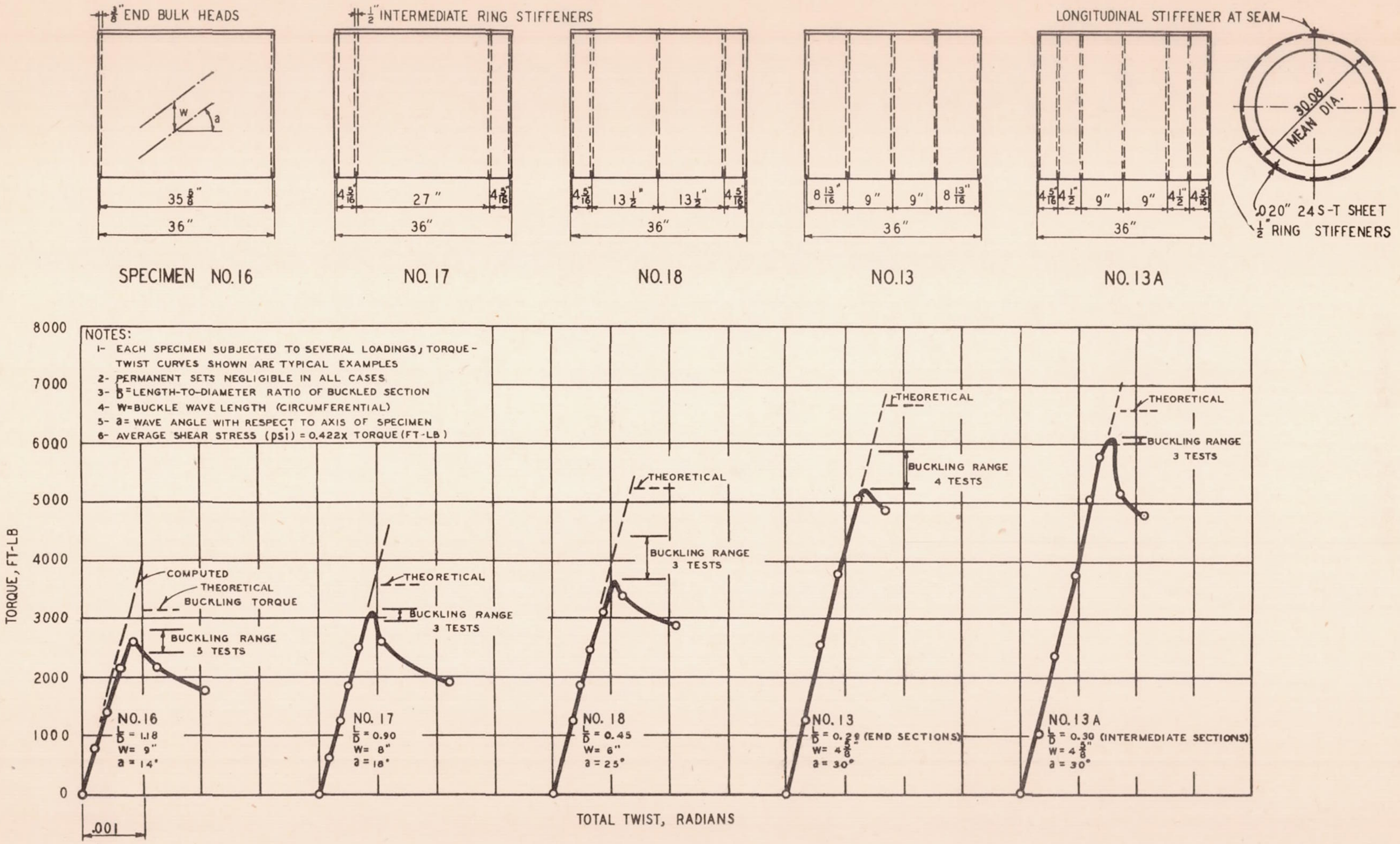


Figure 5 TORSIONAL STRENGTHS OF STIFFENED CIRCULAR CYLINDERS

W-89

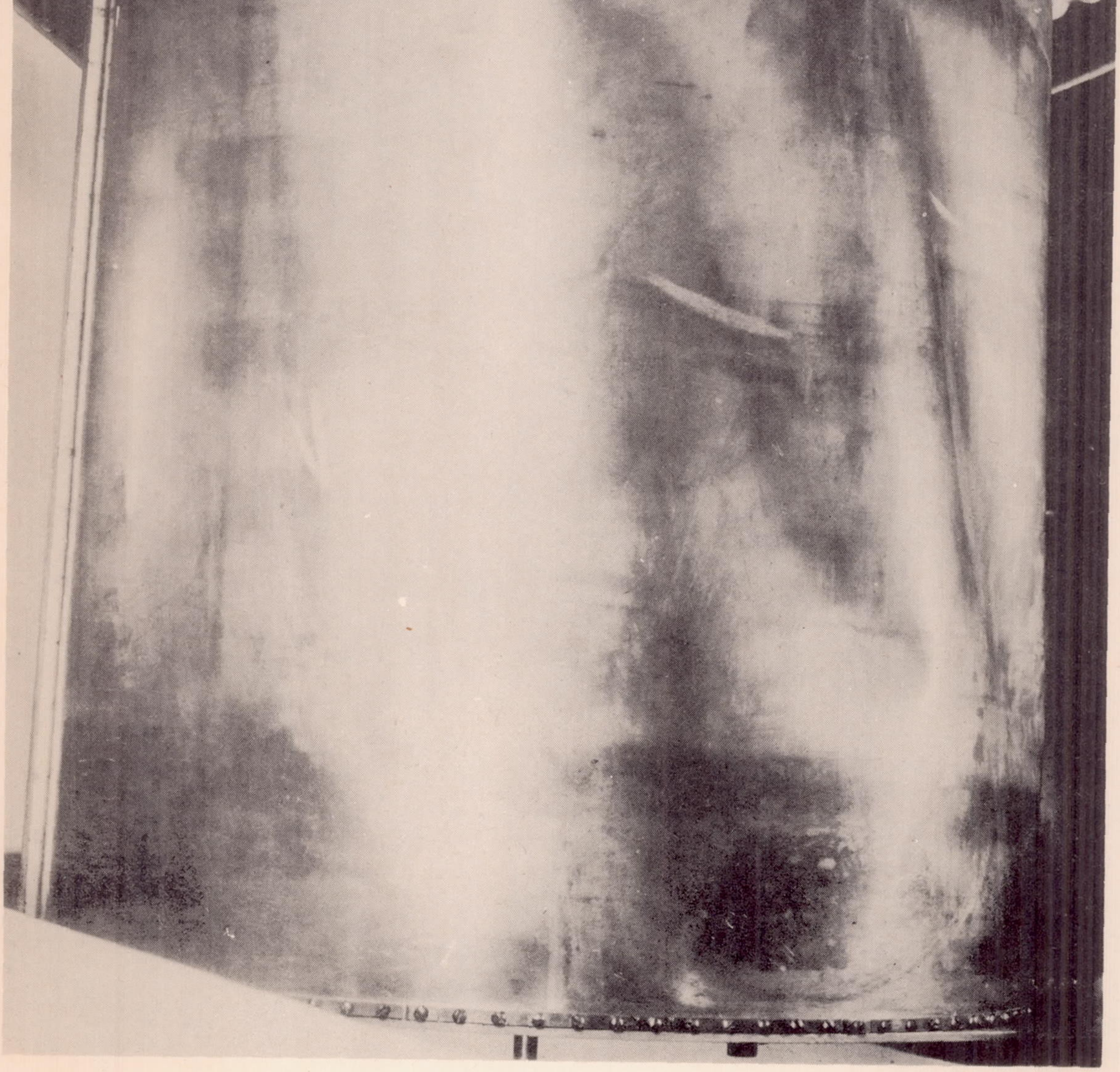


Figure 6. - Failure of specimen No. 16.

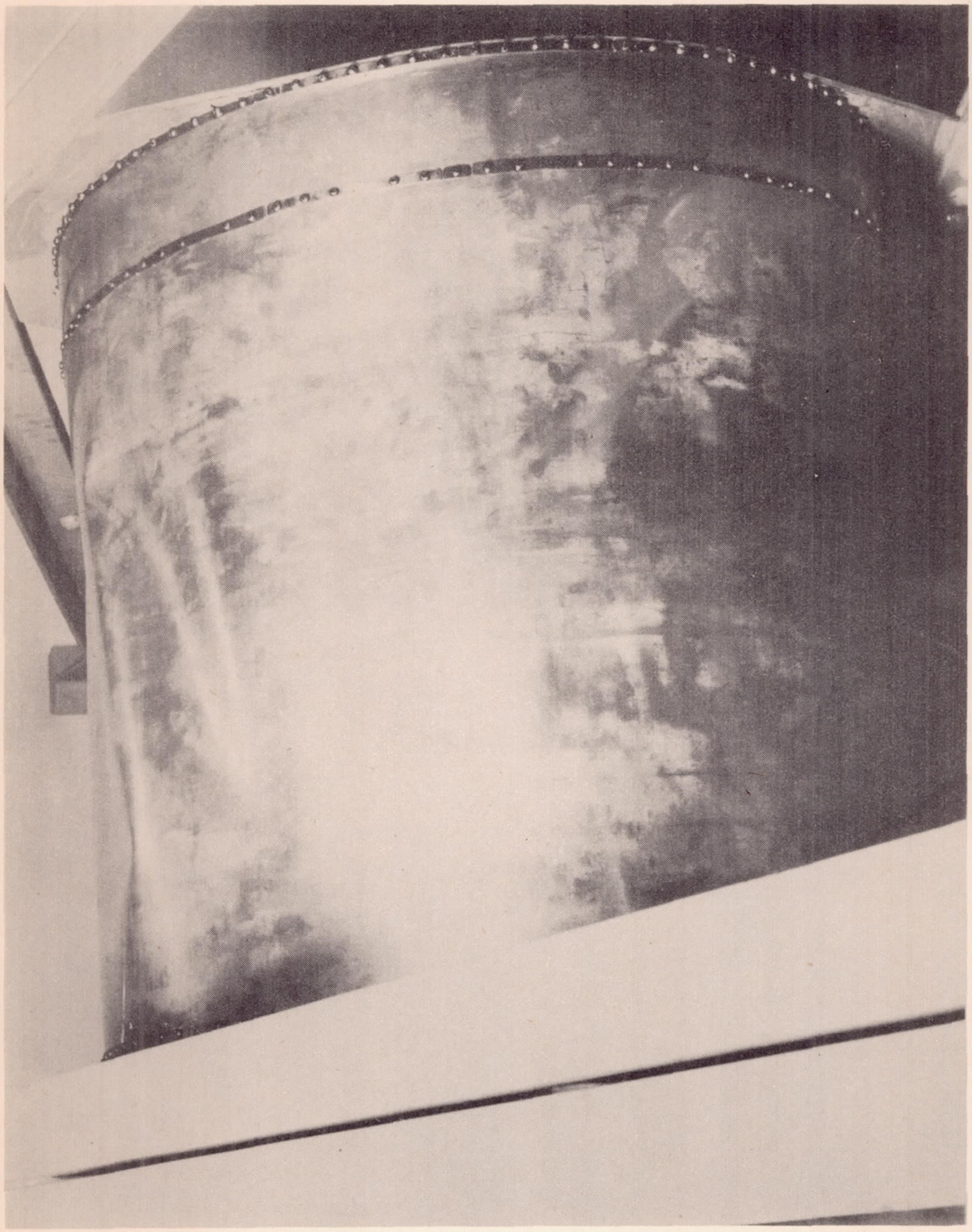


Figure 7. - Failure of specimen No. 17.

w-89

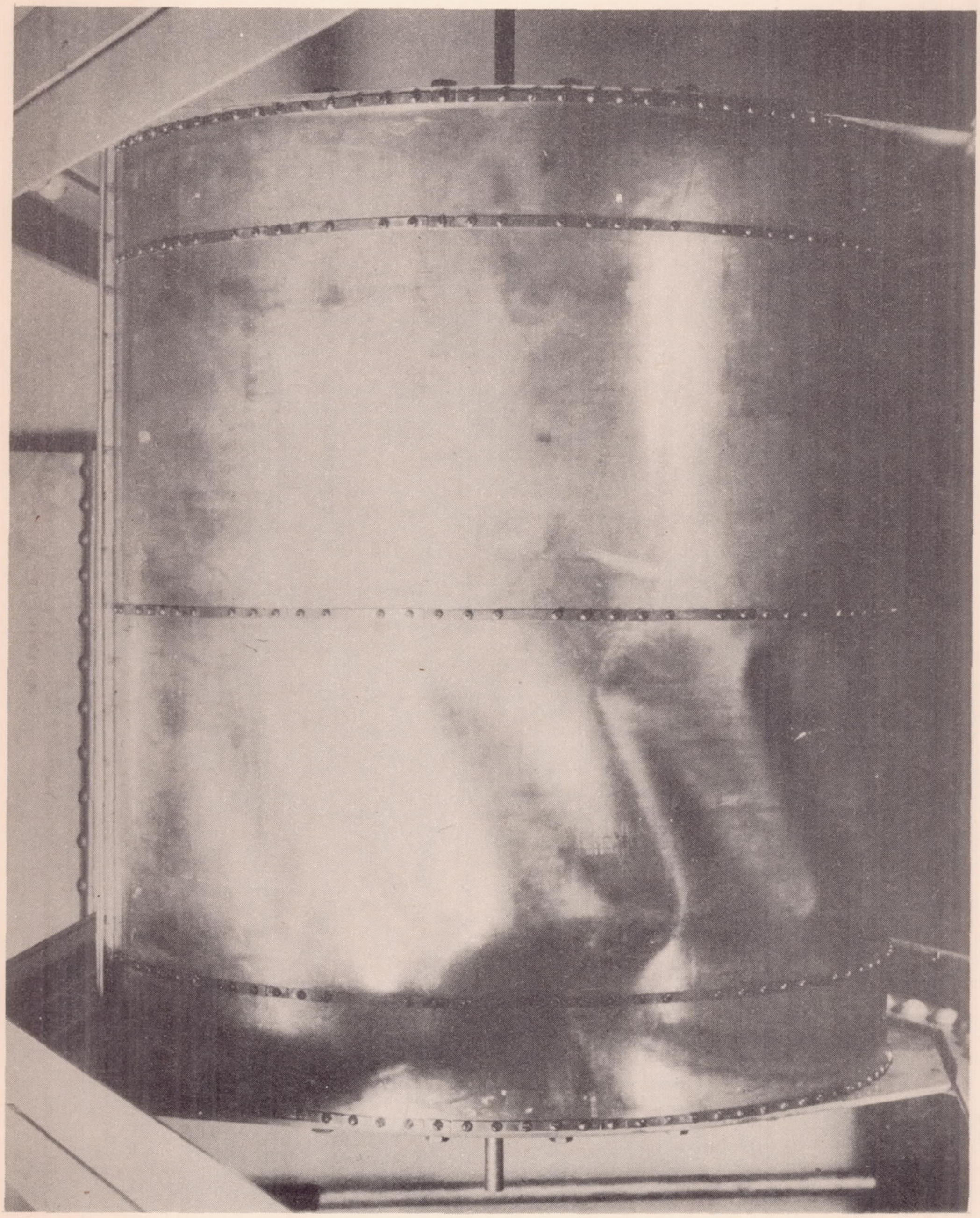


Figure 8. - Failure of specimen No. 18.

w-89

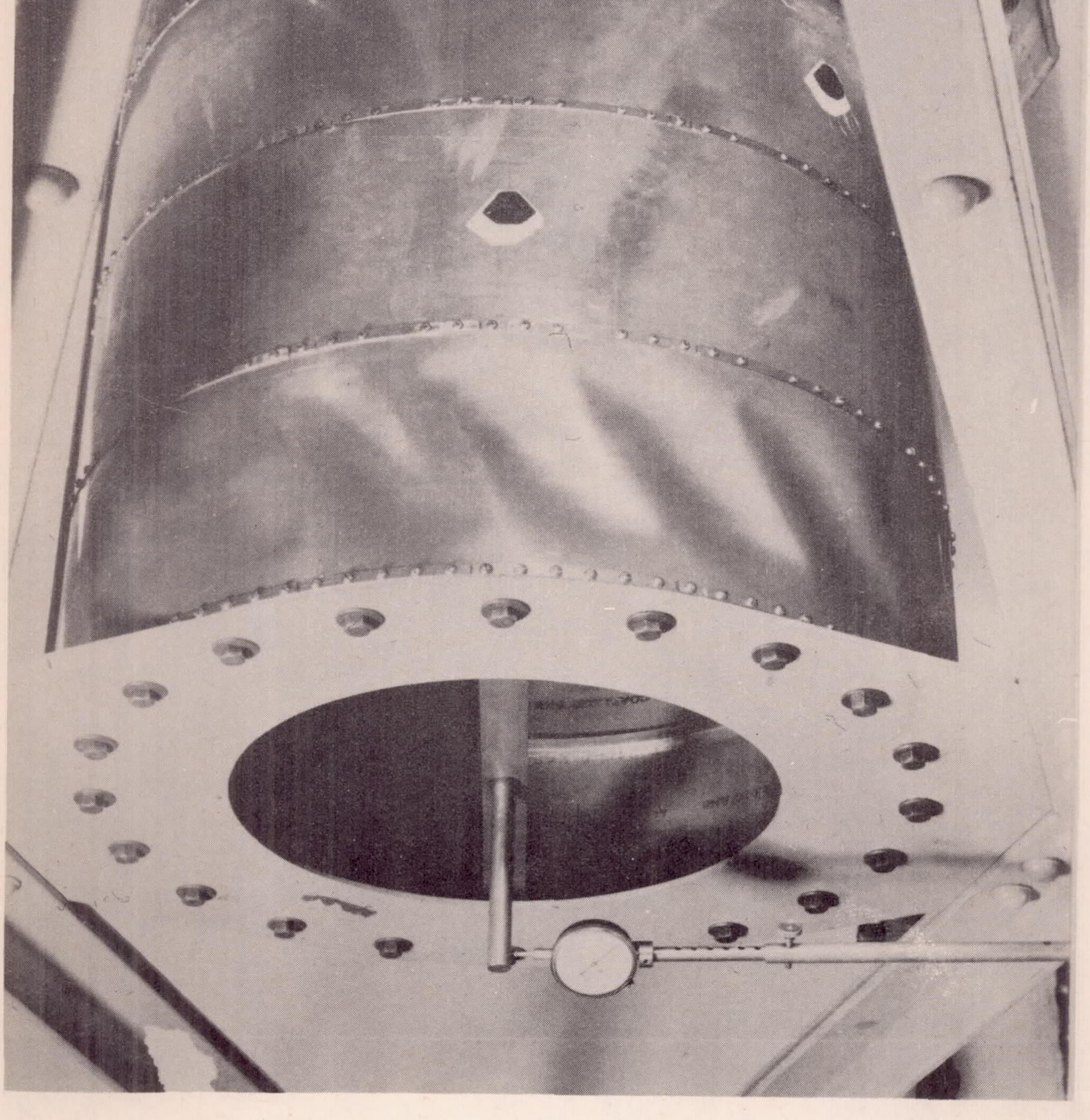


Figure 9. - Failure of specimen No. 13.

W-89

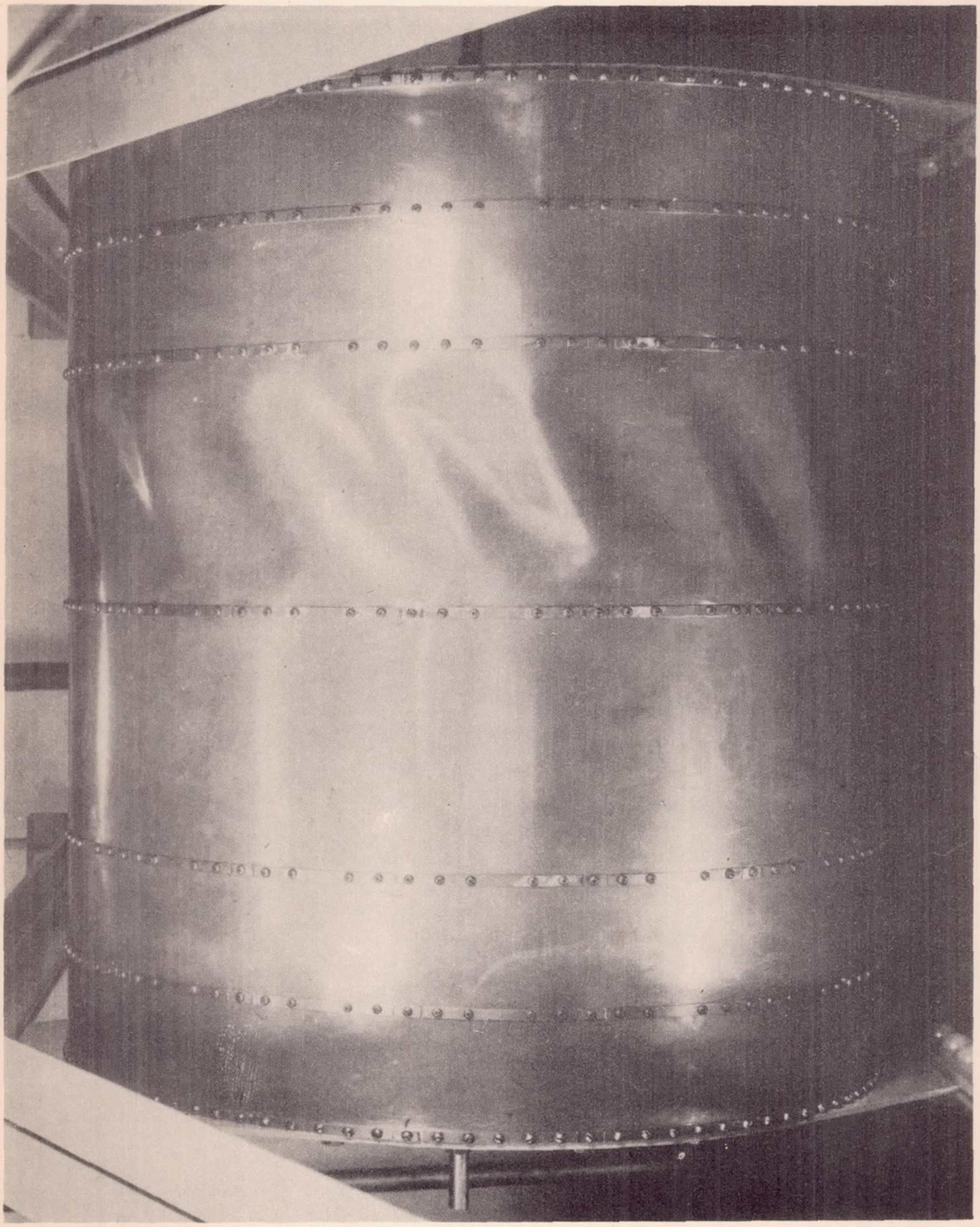


Figure 10. - Failure of specimen No. 13A.

w-89.

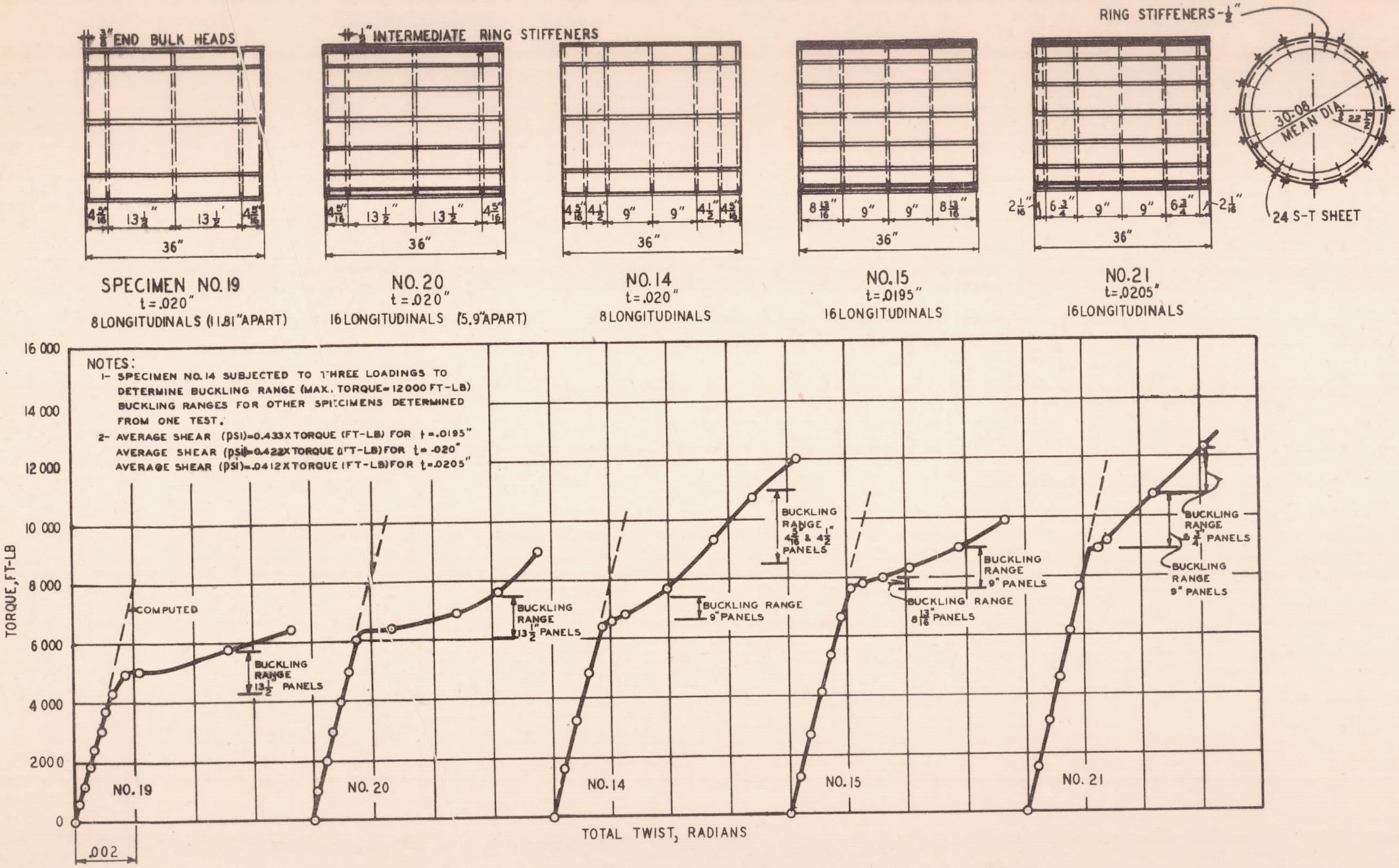
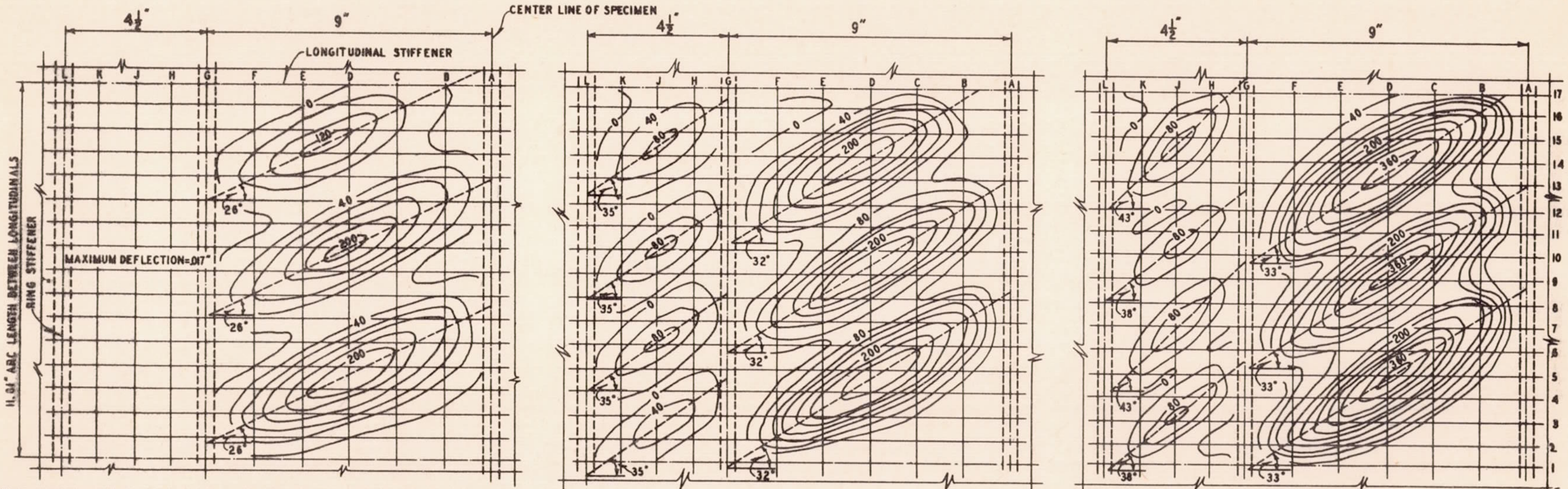


Figure 11.- SHEAR-BUCKLING TORQUES FOR PANELS OF STIFFENED CIRCULAR CYLINDERS



(MEAN BUCKLING TORQUE = 9800 FT-LB) (MEAN BUCKLING TORQUE = 7000 FT-LB)

TORQUE = 9280 FT-LB

TORQUE = 17800 FT-LB

TORQUE = 28000 FT-LB

SPECIMEN NO.14

Figure 12.- DEFLECTION CONTOURS FOR TYPICAL SHEET PANELS
DEVELOPED VIEWS SHOWING RADIAL DEFLECTIONS IN .001 INCHES

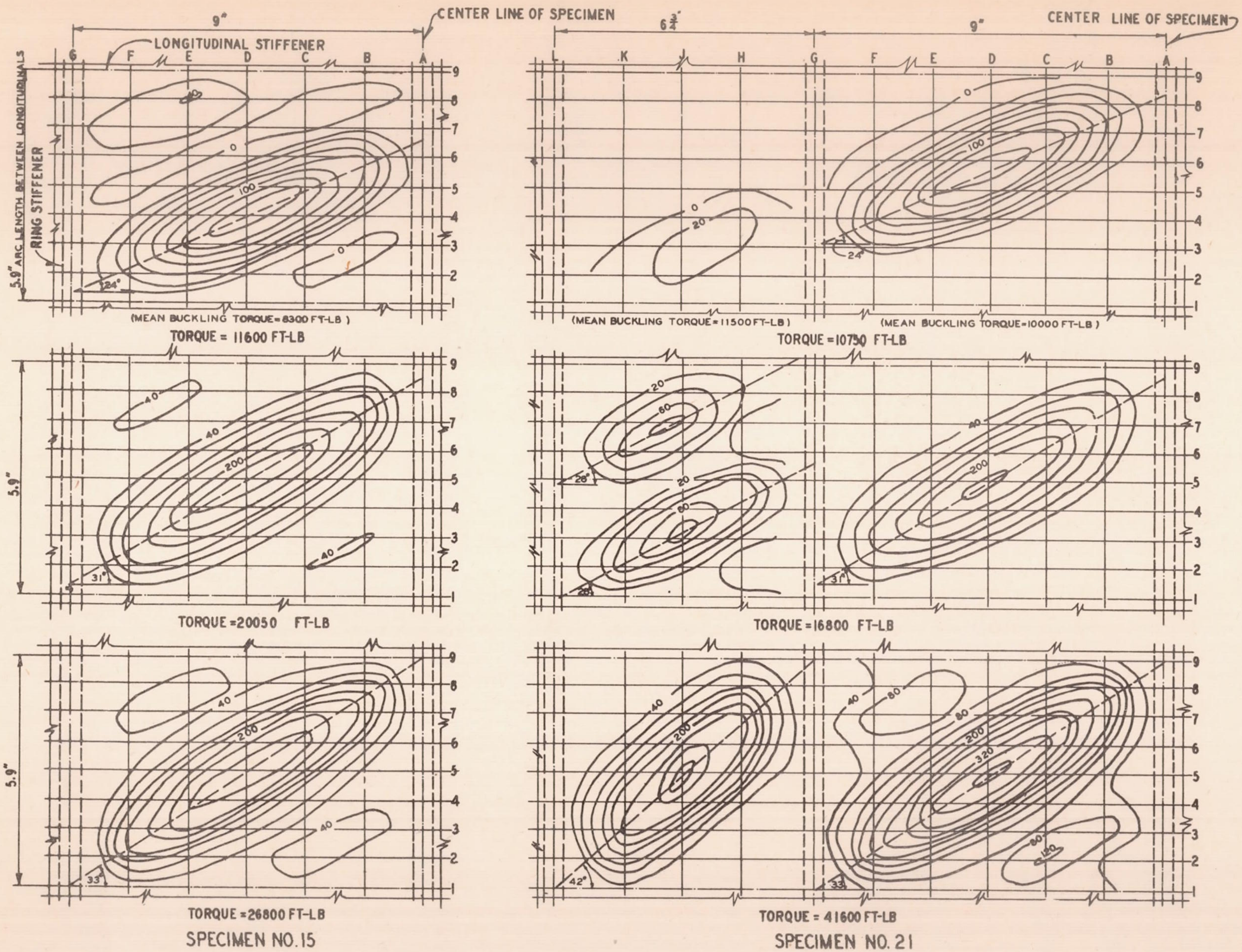


Figure 13.- DEFLECTION CONTOURS FOR TYPICAL SHEET PANELS
DEVELOPED VIEWS SHOWING RADIAL DEFLECTIONS IN .001 IN.

w - 89

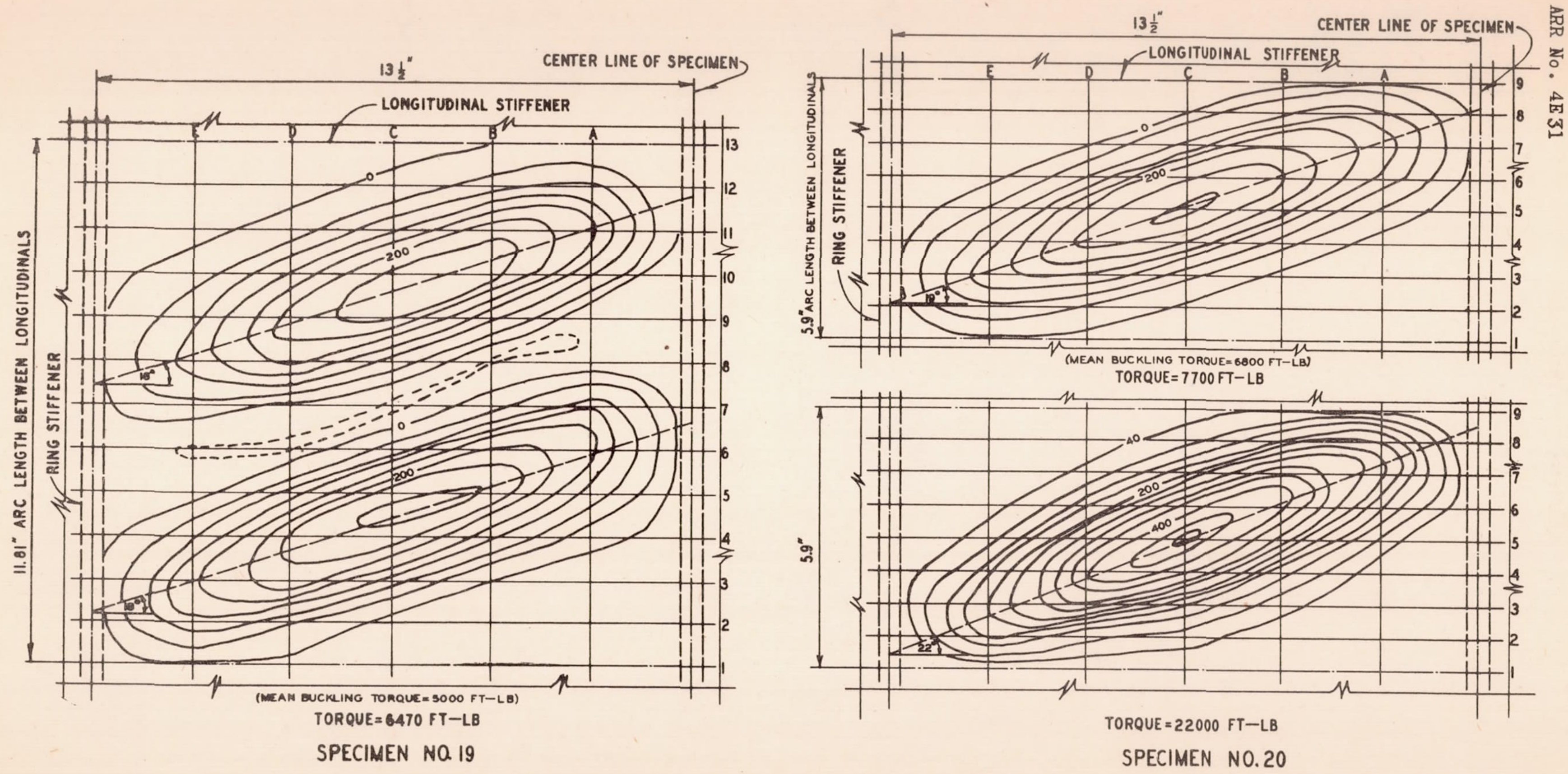


Figure 14.- DEFLECTION CONTOURS FOR TYPICAL SHEET PANELS
DEVELOPED VIEWS SHOWING RADIAL DEFLECTIONS IN .001 IN.

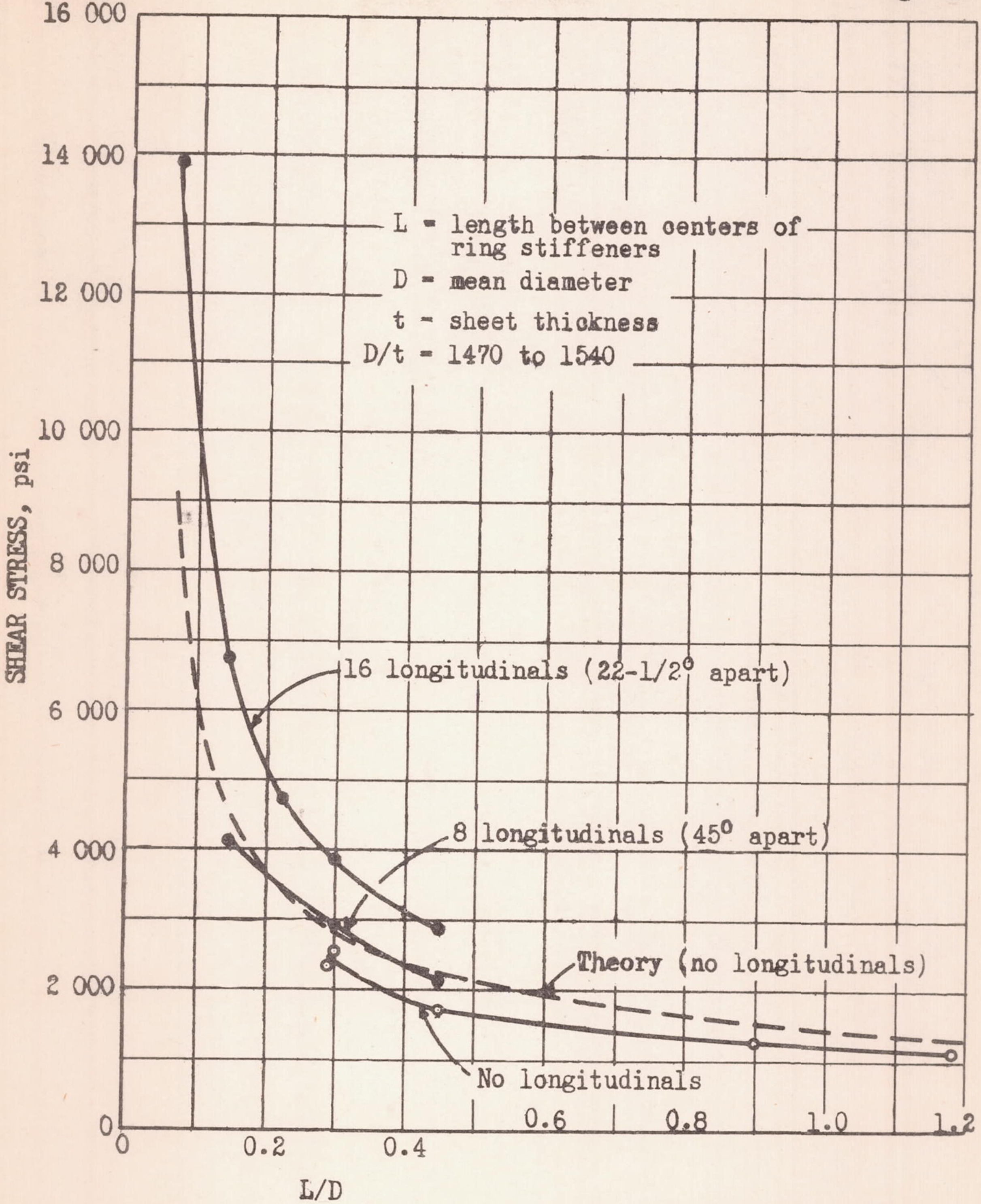


Figure 15.- Shear-Buckling Resistance vs L/D for Different Spacings of Longitudinal Stiffeners

9-6491

24S-T

w-89

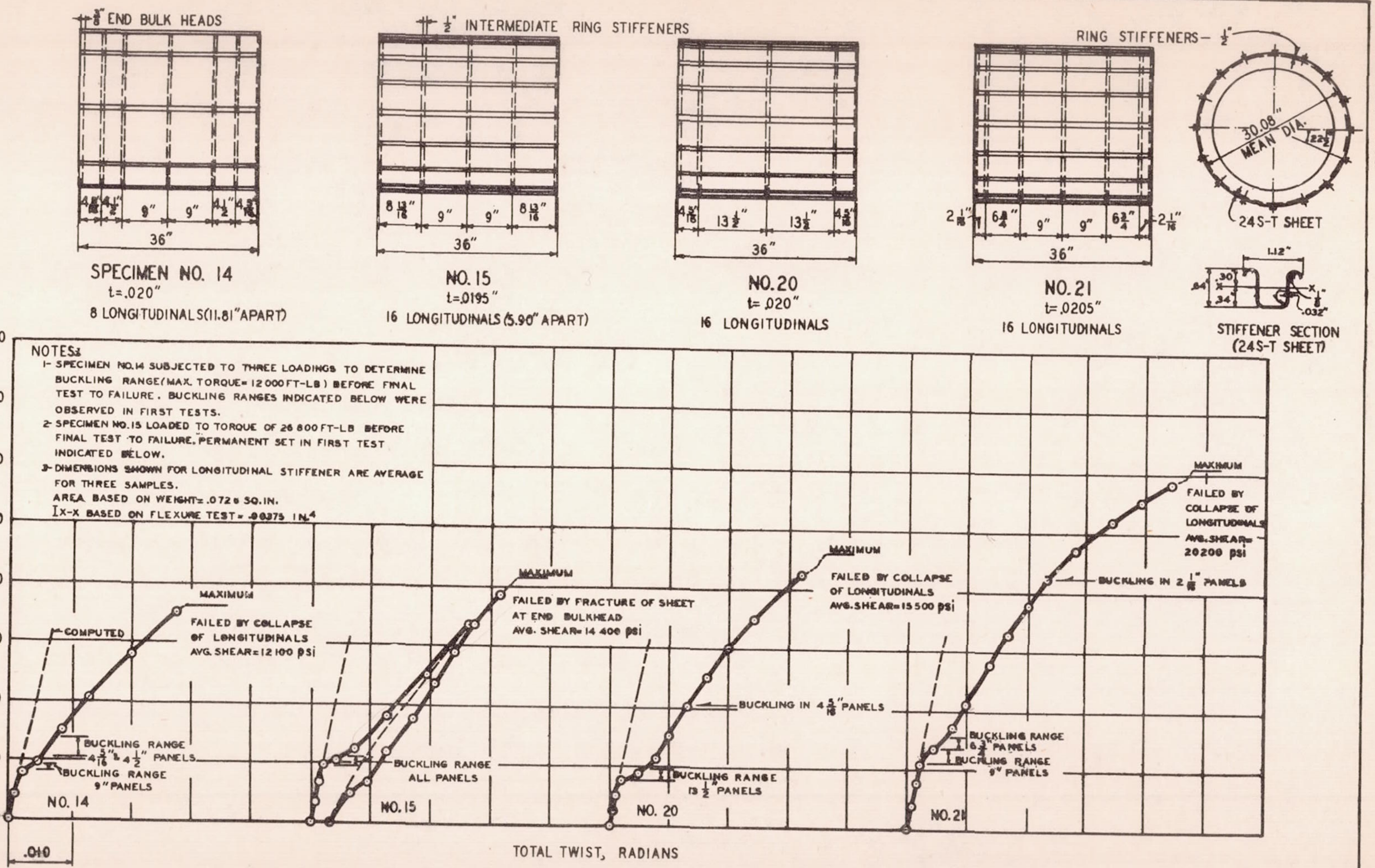


Figure 16.- TORSIONAL STRENGTHS OF STIFFENED CIRCULAR CYLINDERS DEVELOPING TENSION-FIELD ACTION

w-89

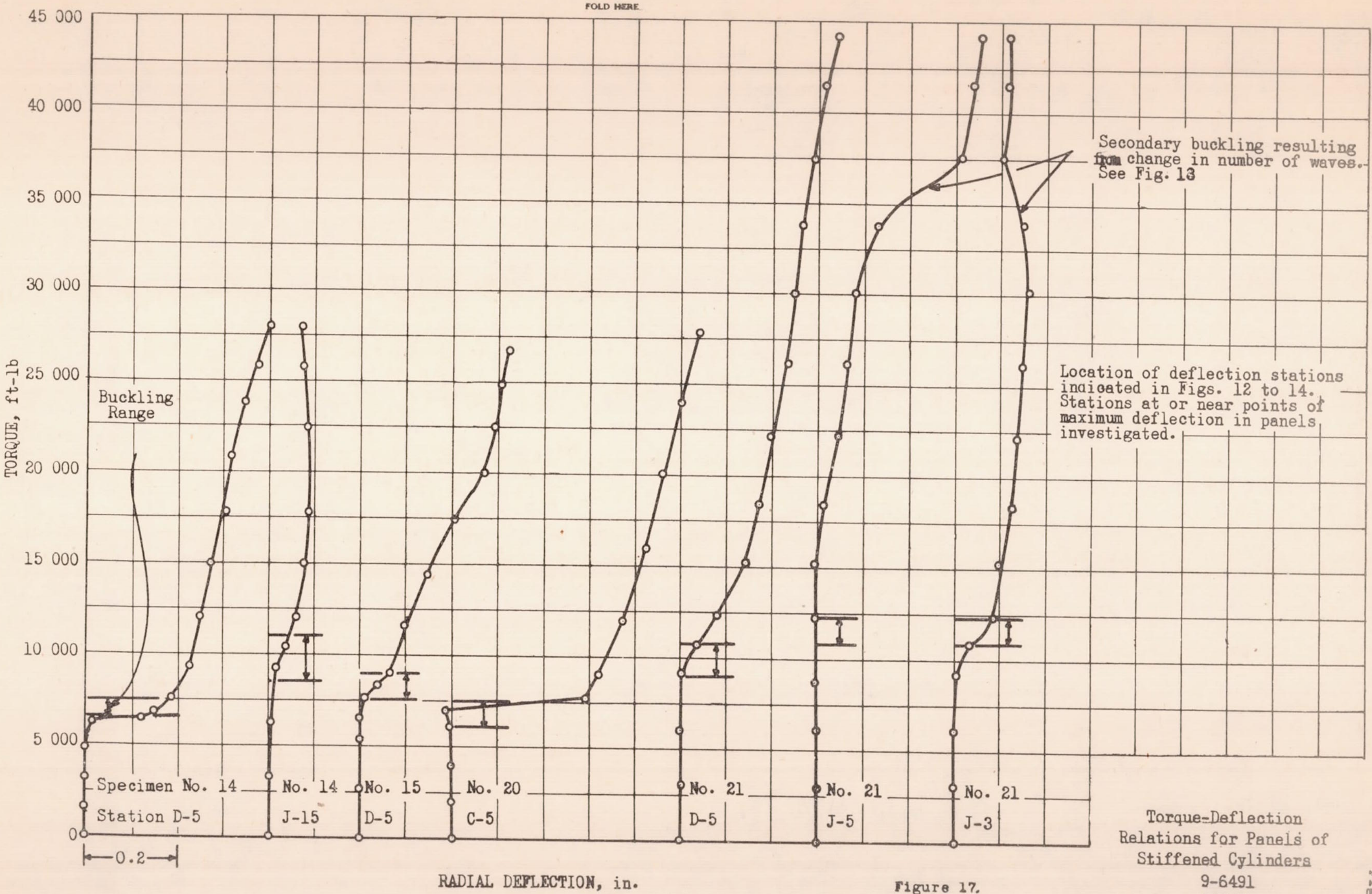


Figure 17.

Torque-Deflection Relations for Panels of Stiffened Cylinders
9-6491
24S-T

w-89

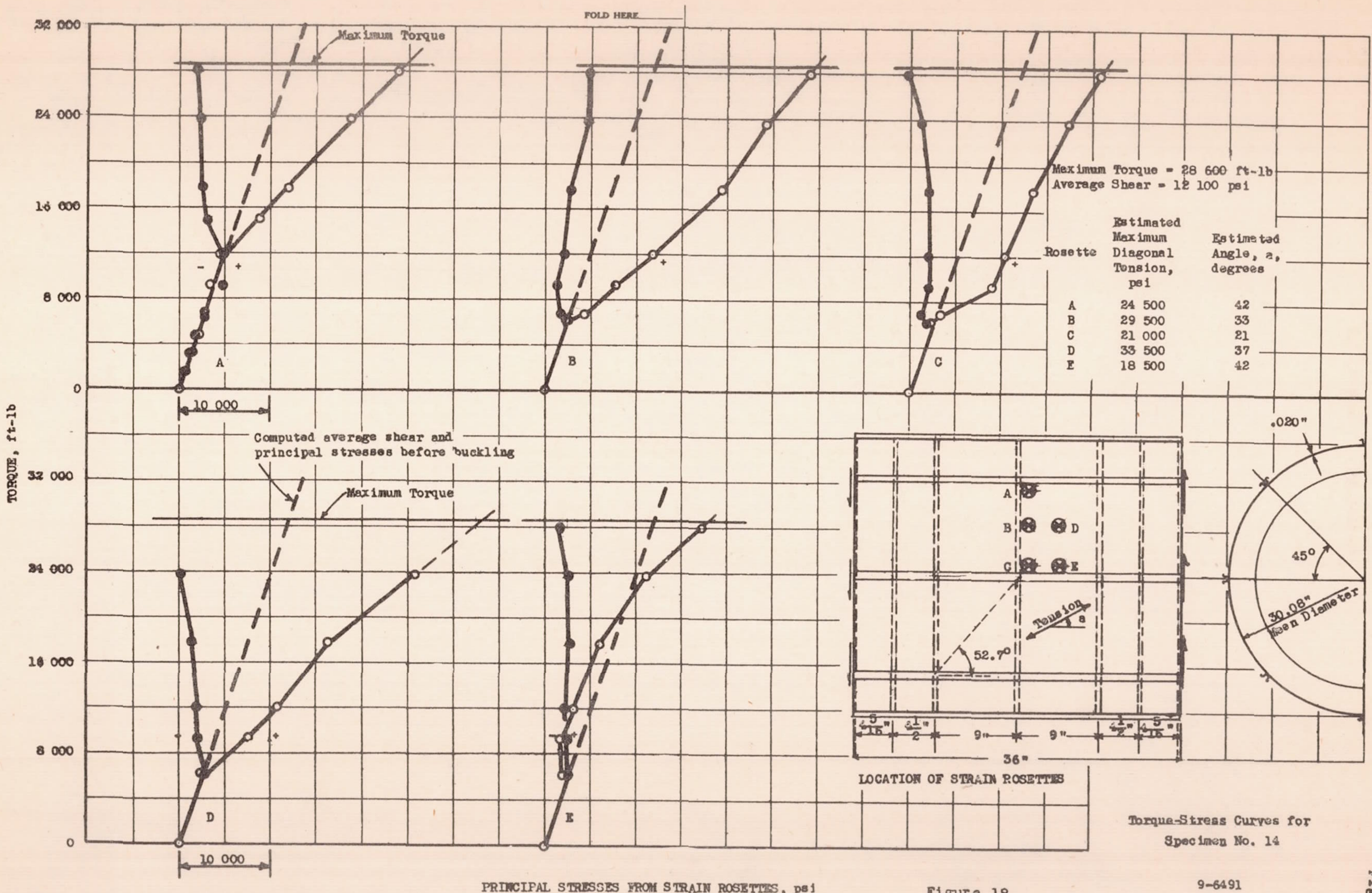


Figure 18.

9-6491
24S-T

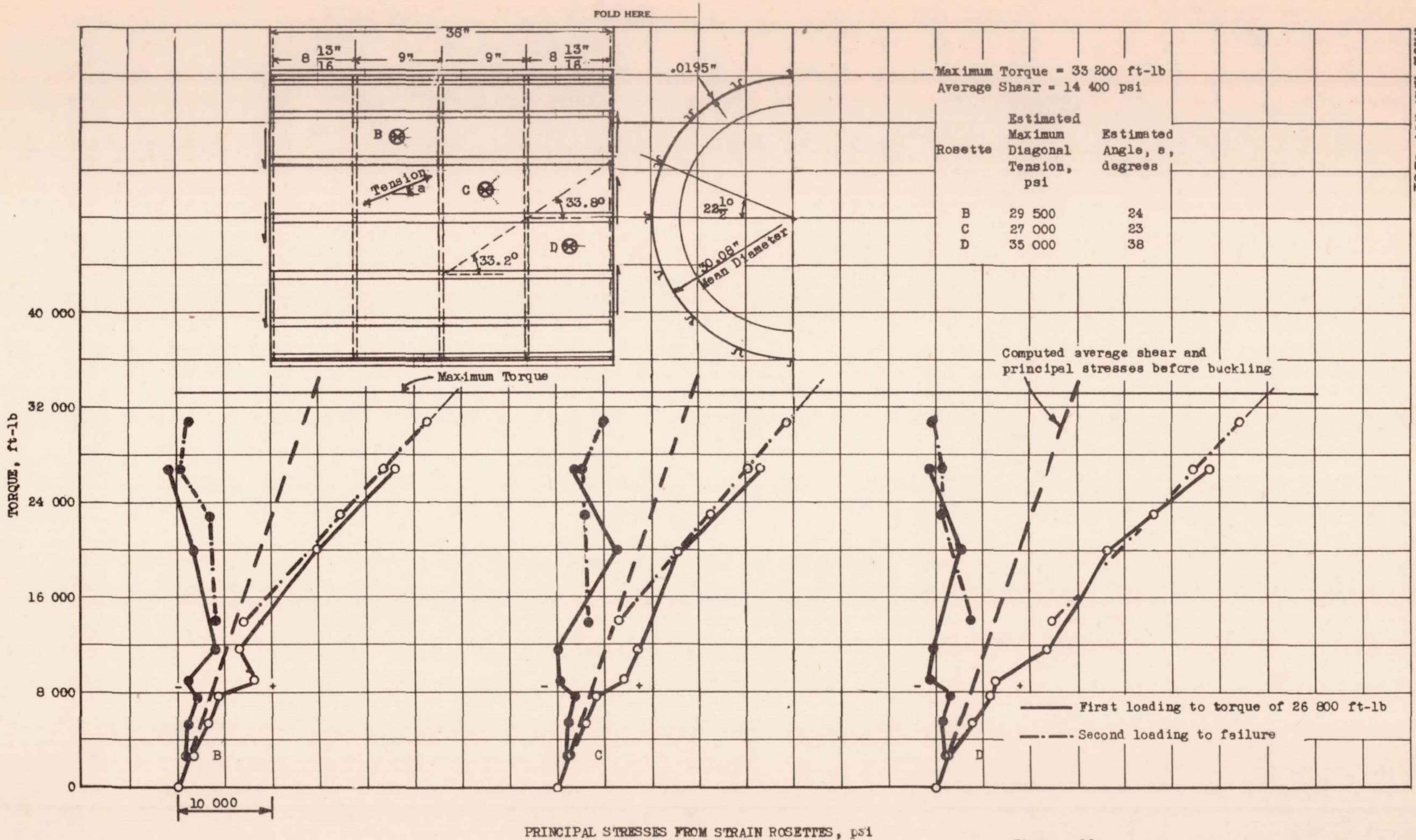


Figure 19. Torque-Stress Curves for Specimen No. 15

9-6491

24S-T

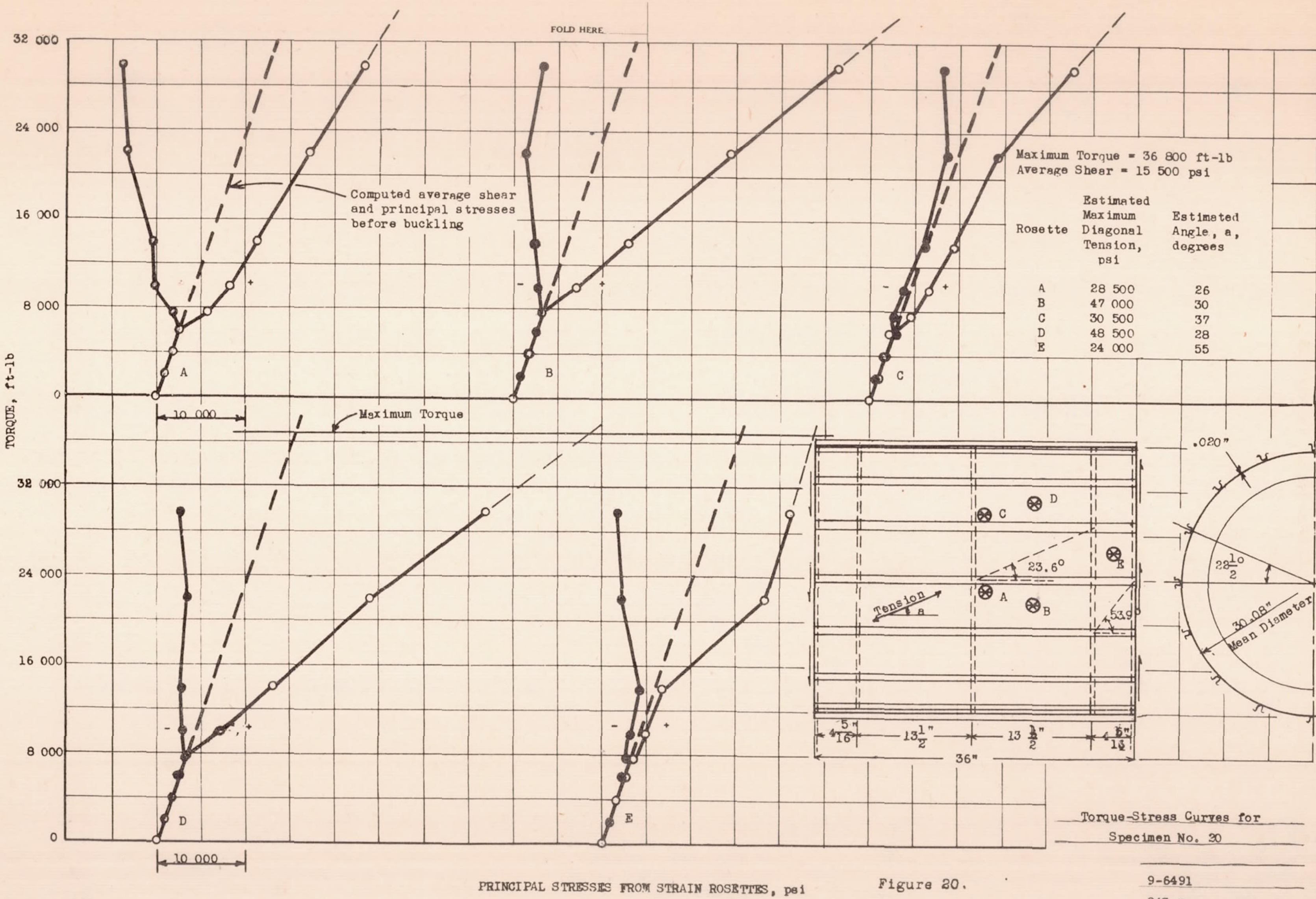


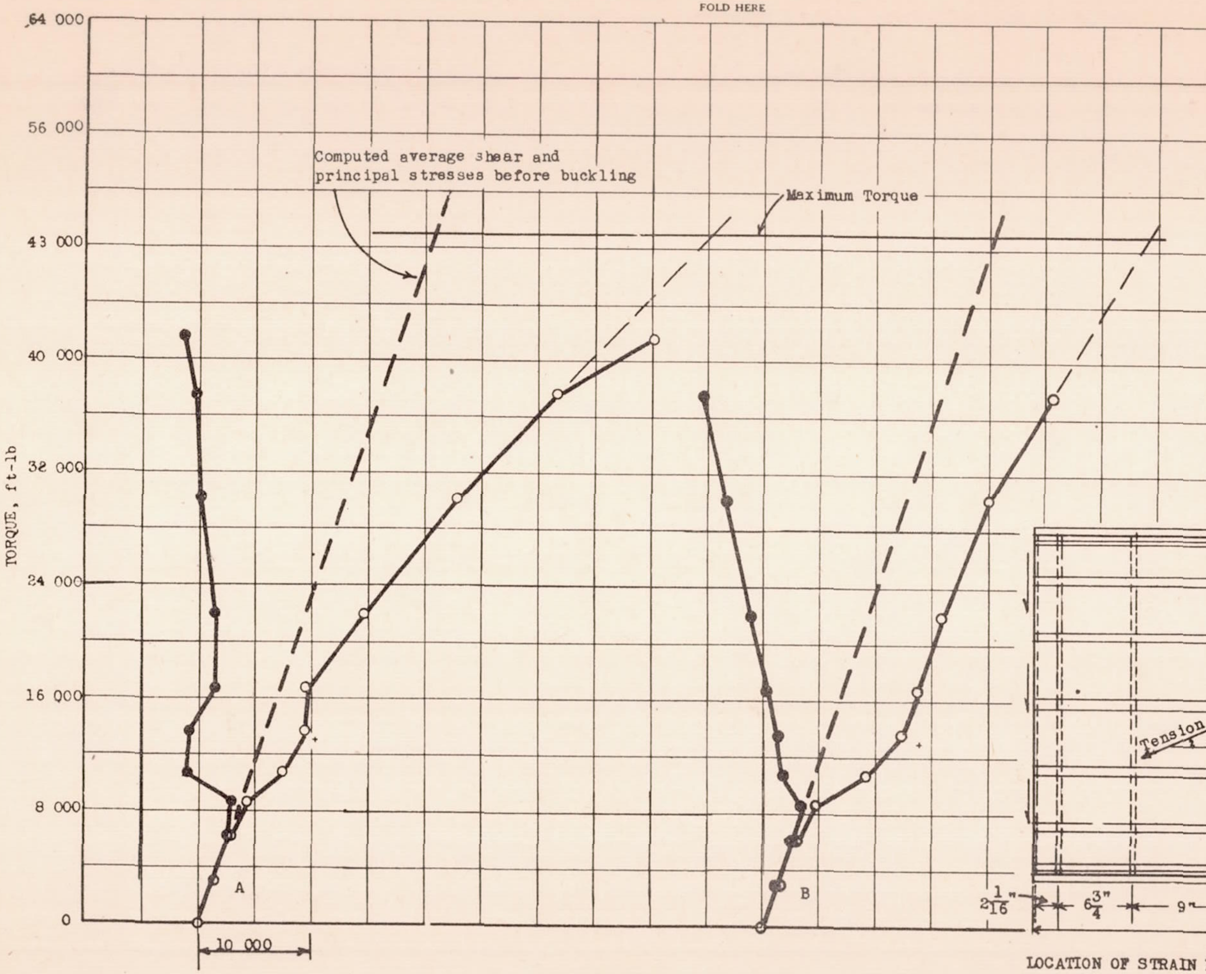
Figure 20.

Torque-Stress Curves for Specimen No. 30

9-6491

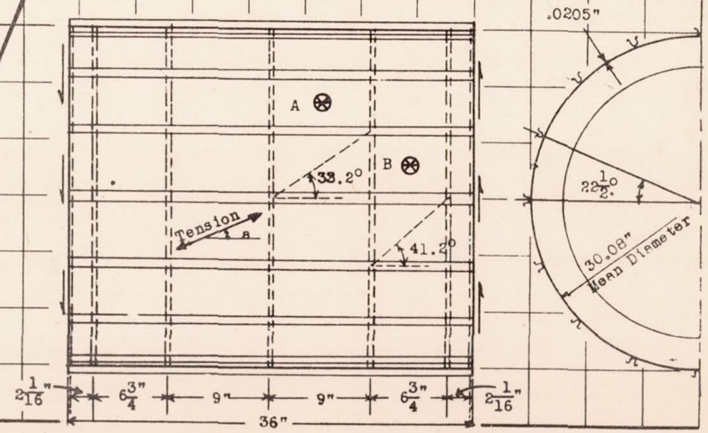
24S-T

FOLD HERE



Rosette	Estimated Maximum Diagonal Tension, psi	Estimated Angle, a, degrees
A	45 500	30
B	34 000	42

PRINCIPAL STRESSES FROM STRAIN ROSETTES, psi



LOCATION OF STRAIN ROSETTES

Figure 21, Torque-Stress Curves for Specimen No. 21

9-6491
24S-T

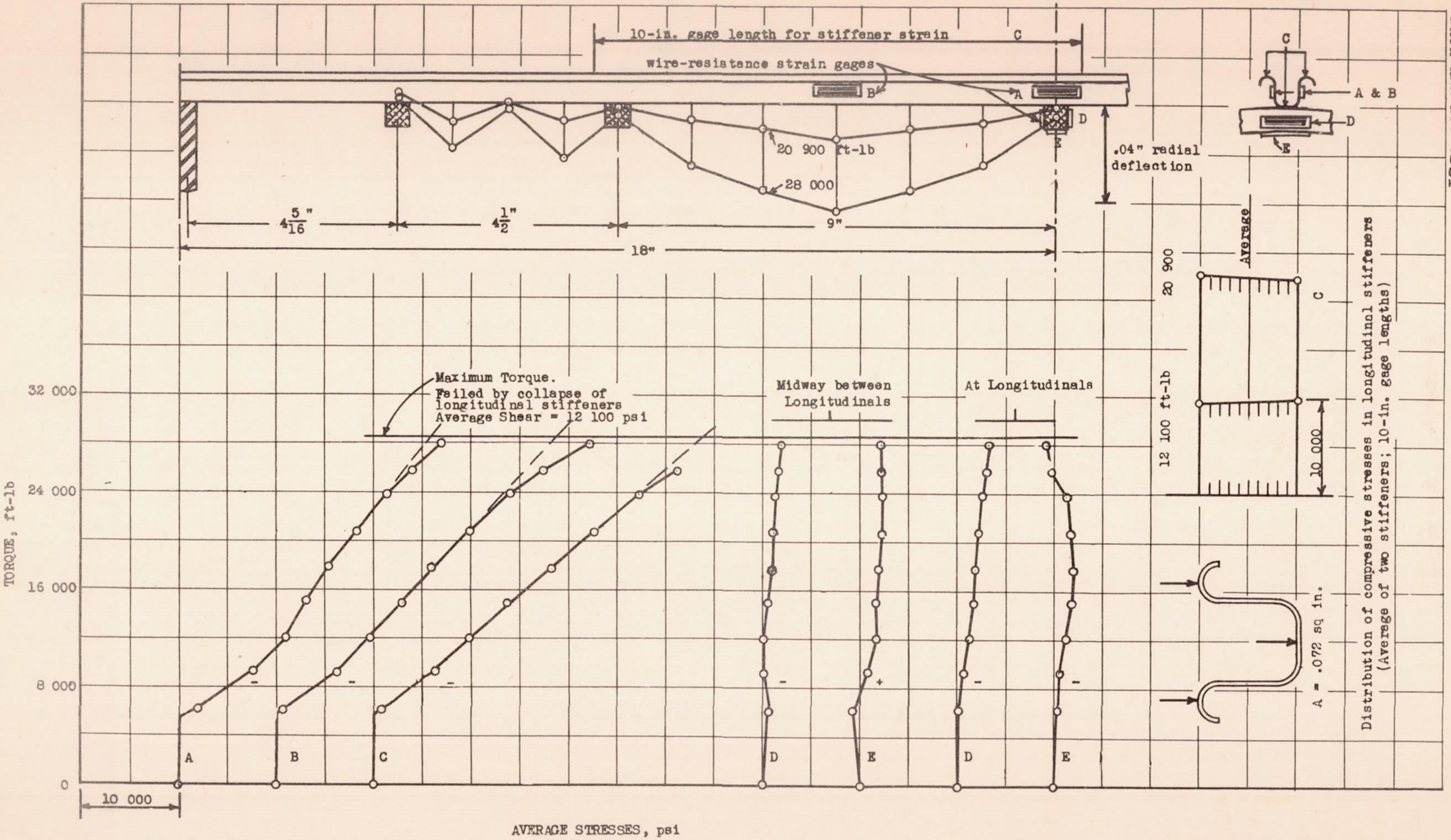


Figure 22.

Deflections shown are average for three stiffeners.
Stresses indicated by Curves A and B are for one stiffener.
Stresses indicated by Curve C are average for two stiffeners.

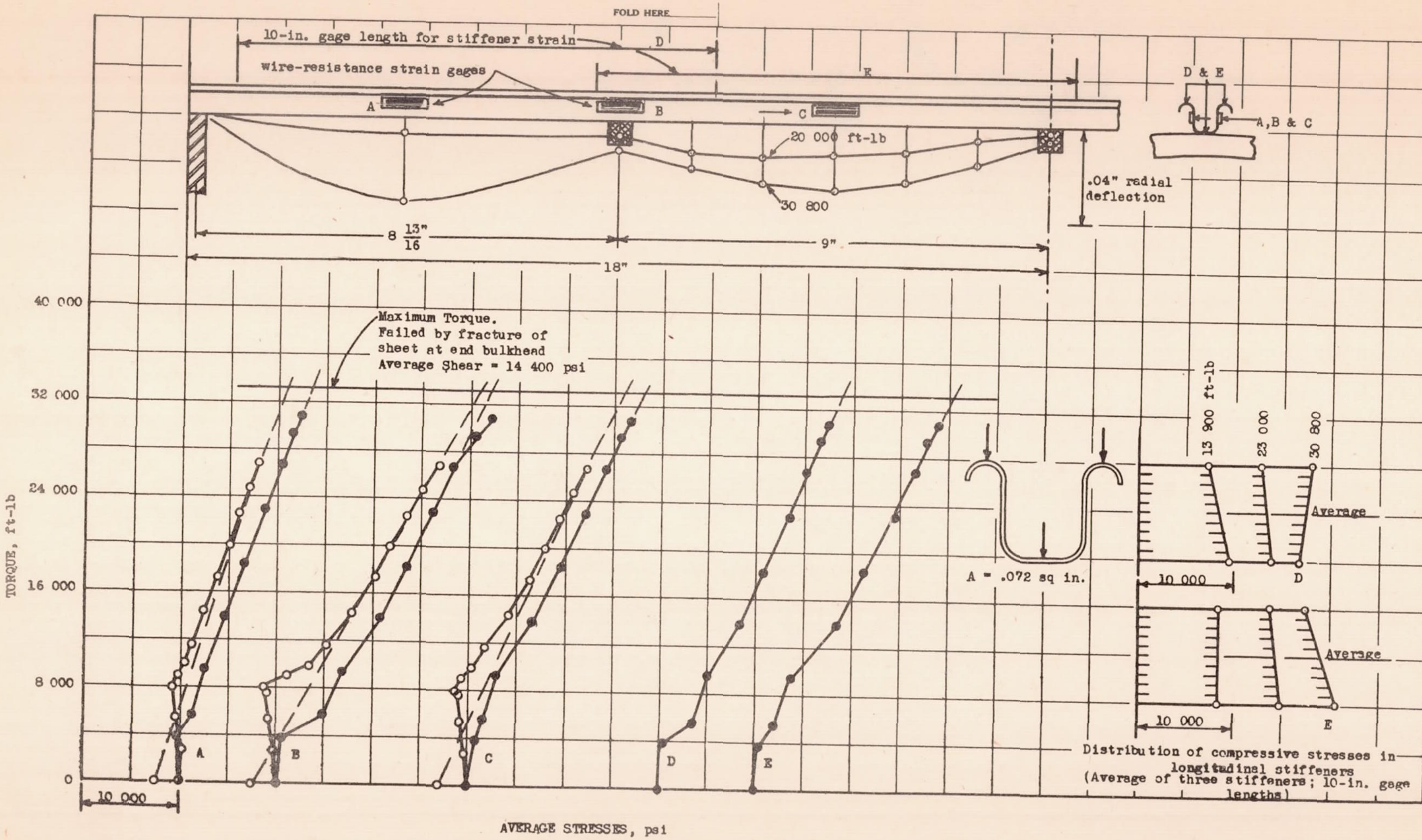
Stiffener Deflections and Stresses in Specimen No. 14

9-6491

24S-T

w-89

NACA ARR No. 4E31



First test to 26 800 ft-lb. o——o
Final test to failure. •——•
Deflections shown are average for three stiffeners.
Stresses indicated by Curves D and E are for three stiffeners.
Stresses indicated by Curves A, B, and C are for one stiffener.

Figure 23. Stiffener Deflections and Stresses in Specimen No. 15

9-6491

24S-T

Fig. 23

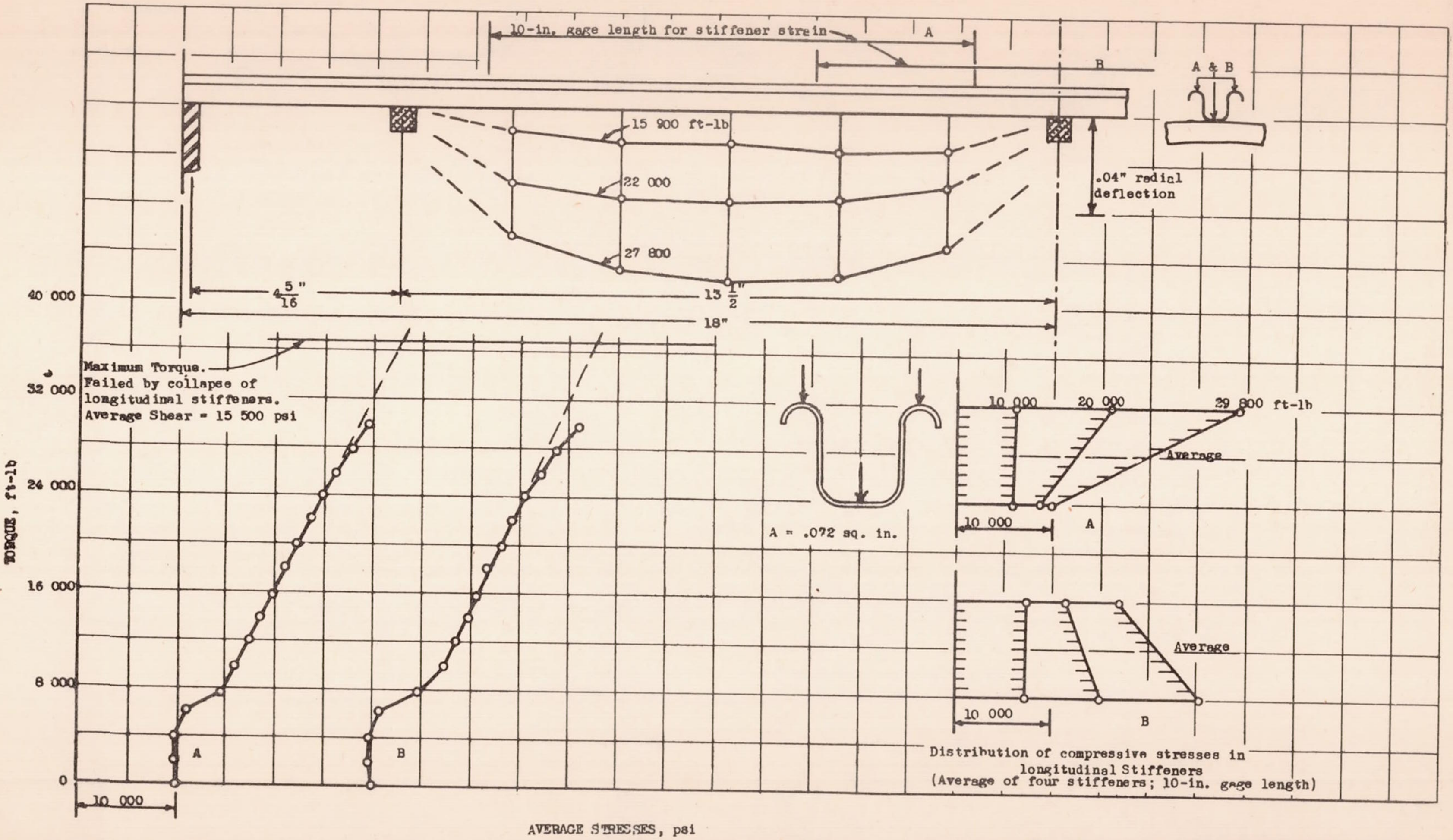
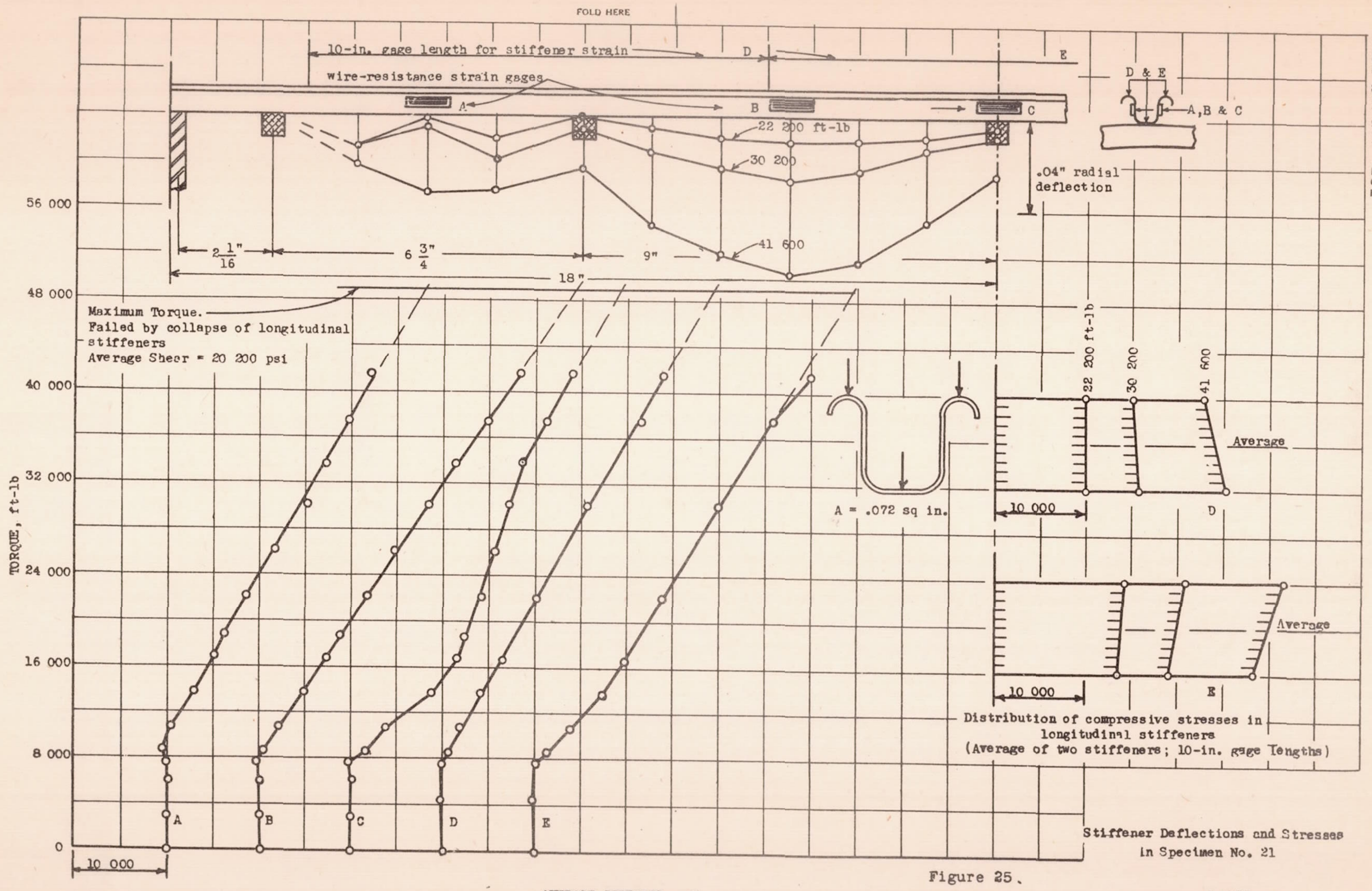


Figure 24. Stiffener Deflections and Stresses in Specimen No. 20

Deflections shown are average for three stiffeners
Stresses indicated by Curves A and B are for four stiffeners

9-6491
24S-T

w-89



Stiffener Deflections and Stresses in Specimen No. 21

Figure 25.

9-6491

24S-T

Deflections shown are average for three stiffeners.
Stresses indicated by Curves A, B and C are for one stiffener
Stresses indicated by Curves D and E are for two stiffeners.

3641 6M-9-42 PRINTED IN U. S. A. 32957

W-89

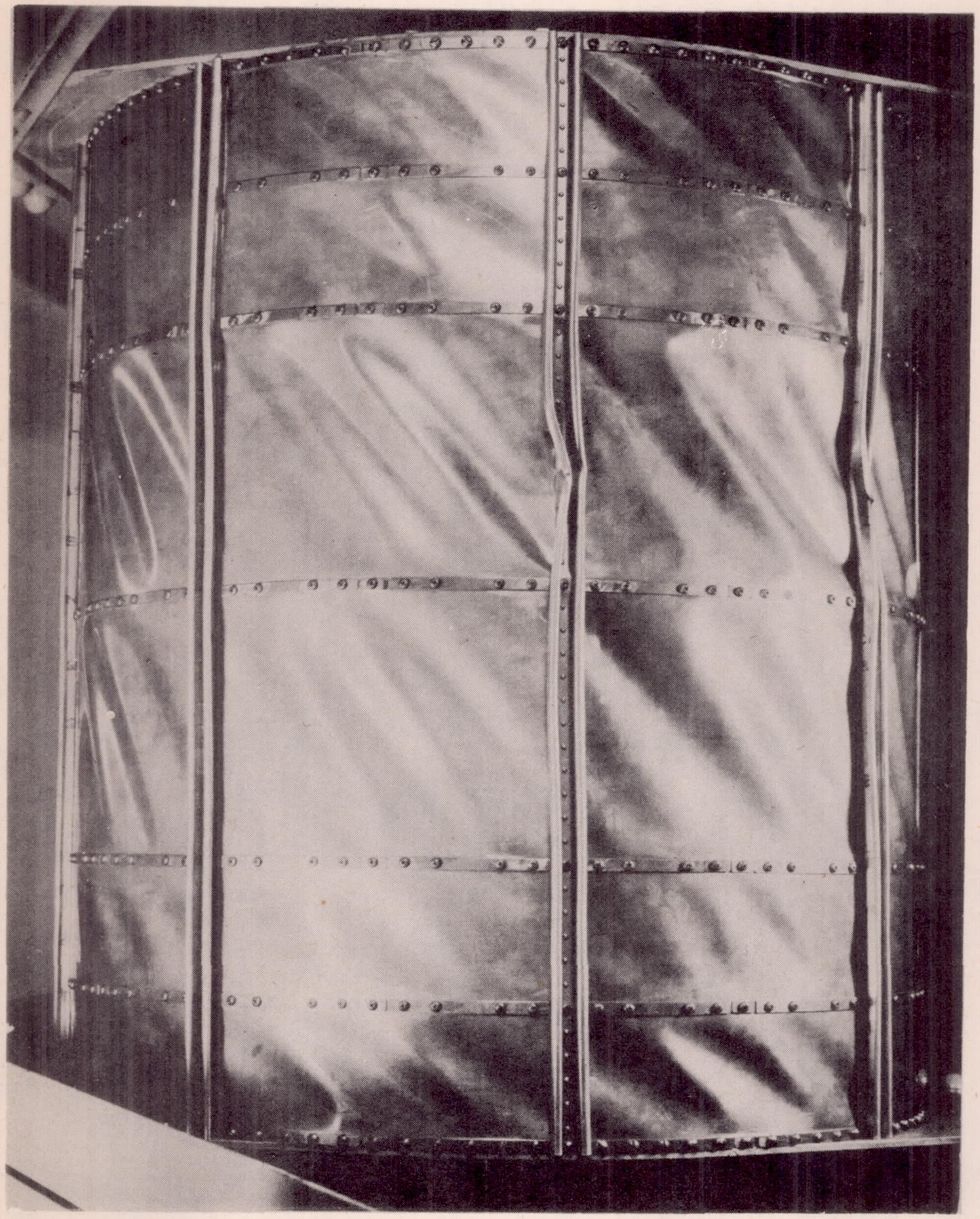


Figure 26. - Failure of specimen No. 14.

w-89

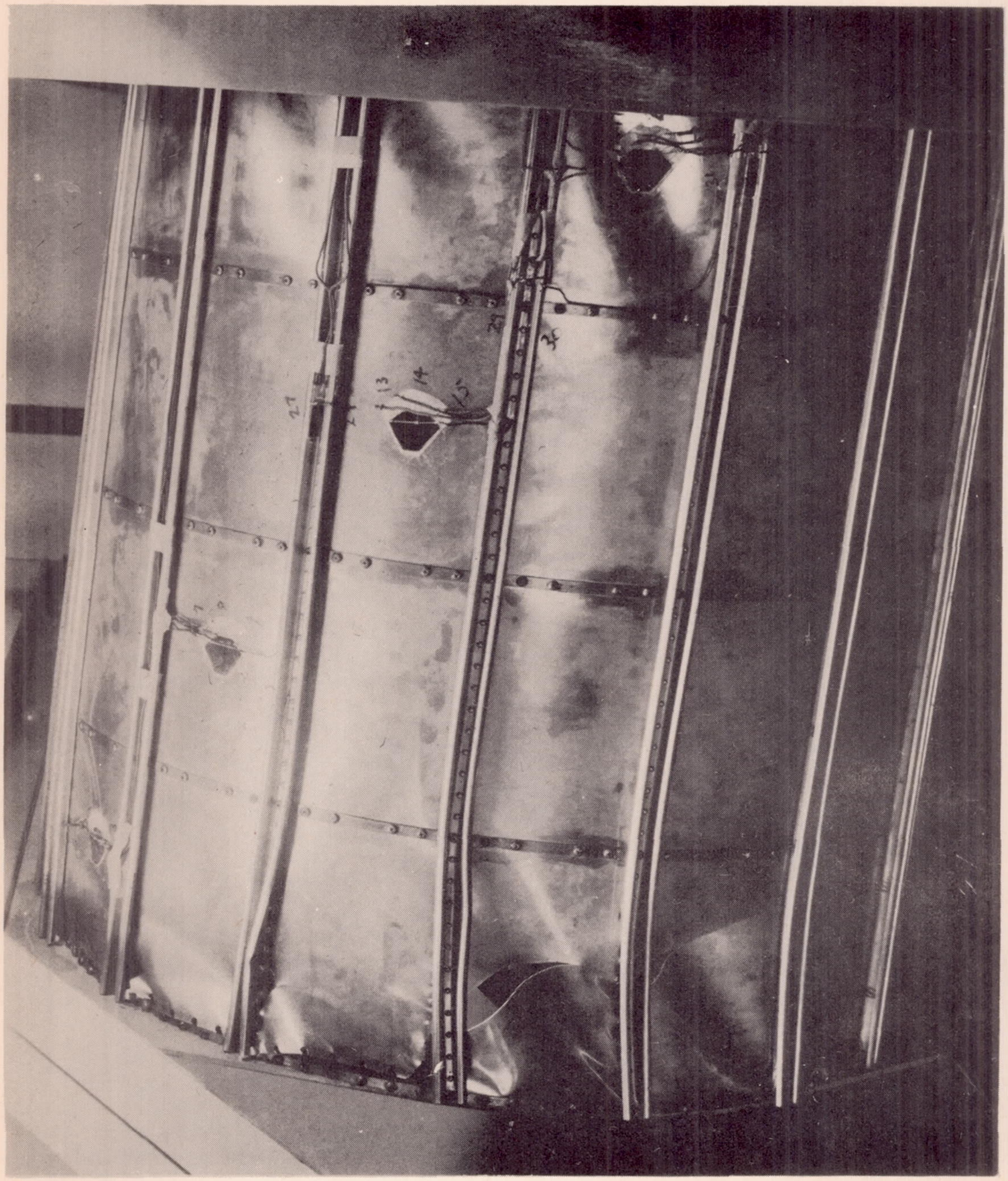


Figure 27. - Failure of specimen No. 15.

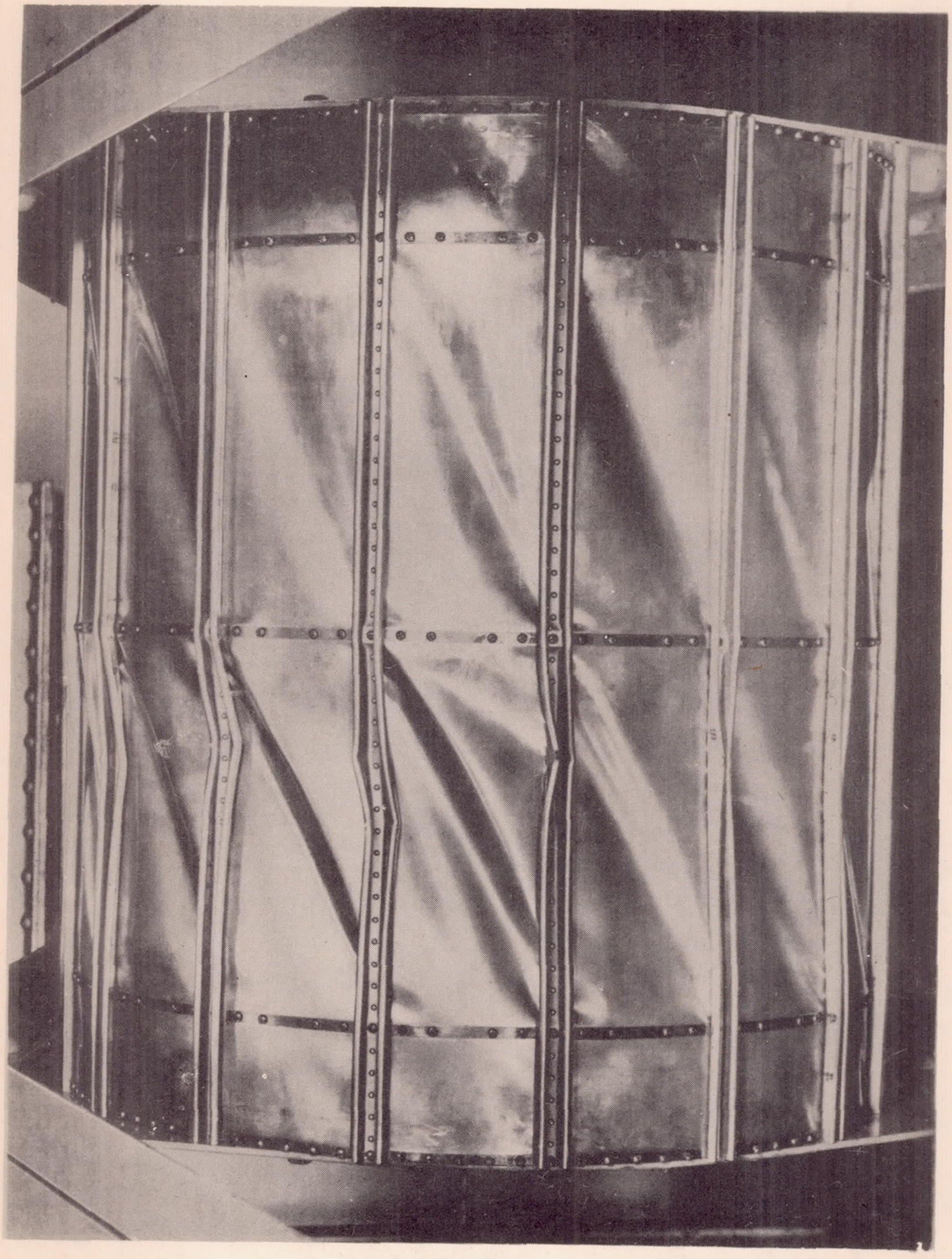


Figure 28. - Failure of specimen No. 20.

w-89

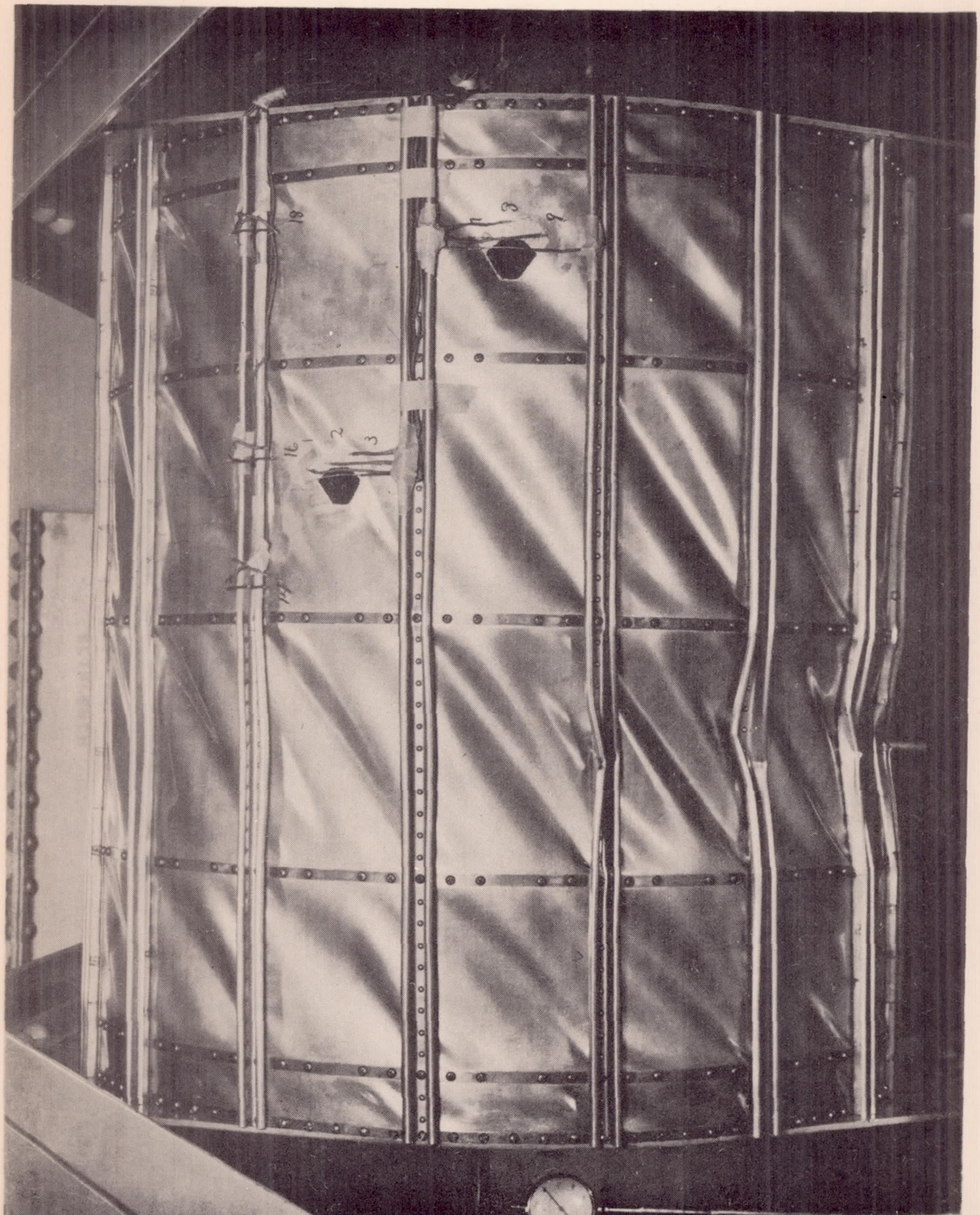


Figure 29. - Failure of specimen No. 21.

w-89

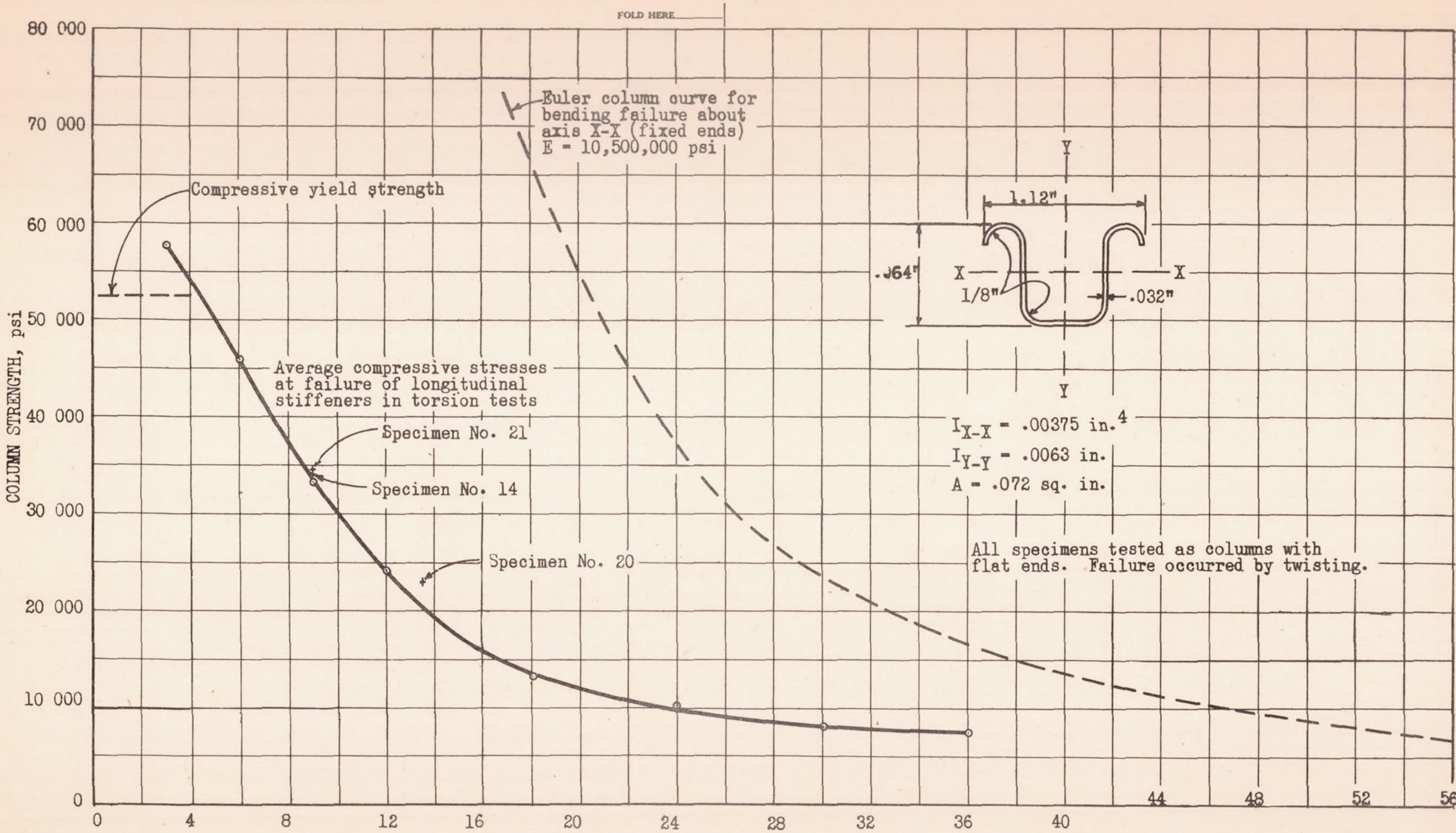


Figure 30. Relation Between Column Strength and Length of Longitudinal Stiffeners
9-6491
24S-T